

Primal Bach: A Number Secret Revealed

Transcription of presentation with slides for the 2025 Pipe Organ Centennial
by UF Professor of Music, emeritus Willis Bodine
Friday, January 25, 2025, 3:00 pm in MUB 101

[title slide with Sieve of Eratosthenes]

Primal Bach

A NUMBER SECRET REVEALED

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UF Pipe Organ Centennial
Friday, January 24, 2025, at 3:00 pm in Music Building 101

	2	3	4	5	6	7	8	9	10	Prime numbers
11	12	13	14	15	16	17	18	19	20	
21	22	23	24	25	26	27	28	29	30	
31	32	33	34	35	36	37	38	39	40	
41	42	43	44	45	46	47	48	49	50	
51	52	53	54	55	56	57	58	59	60	
61	62	63	64	65	66	67	68	69	70	
71	72	73	74	75	76	77	78	79	80	
81	82	83	84	85	86	87	88	89	90	
91	92	93	94	95	96	97	98	99	100	
101	102	103	104	105	106	107	108	109	110	
111	112	113	114	115	116	117	118	119	120	

Jon Swett introduction) Welcome, everybody. Dr. Ellis is tying up some wisdom in the auditorium. She asked me if I could introduce the next speaker. I'm very, very glad to do so. I think it holds a fond place in my heart as one of his last organ students in 2003. So, I think you all know, but it's welcome to host Willis Bodine. [applause]

(WRB begins) This is very nostalgic for me in so many ways. This [indicates Room 101] was a band hall when the building was built and it has now become -- since they got money [laughter] -- it has become, I think, a very acceptable recital hall for the School of Music. The school has big plans.

Let's go back, let's go back before January 20th. Let's go back before cell phones. Let's go back before television, before radio, before vaccinations, before the discovery of microbes, before the discovery of electricity. Let's go back before, shall I say, the time when people got involved in machines. When they had time to do things. And one of the things they did was play games -- they had lots of time. They played games with numbers. They played games with letters. And so it was part of culture to do imaginative things with numbers.

Raise your hand if you know what I'm doing: [counts aloud] **2, 4, 6, 8, 10, 12, 14, 16, 18** -- I got almost everybody. This is fourth grade. Let's pick it up. Raise your hand again, if you can tell what I'm doing: [counts] **7, 14, 21, 28**. Quicker response. That's very good.

Here we go. What am I doing? [counts again] **2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31** -- okay, I think I've got a good response. That's almost as many as got the sequence of twos [laughter] or of the sequence of

sevens that I did. Of course this is the **sequence of primes**. Today (I am told) children begin to incorporate primes in their learning of arithmetic very early. I never heard about prime numbers when I was in elementary school, or what I called junior high and you call it middle school. I never heard about it in high school, and thanks to my conservatory within a university [*The University of Texas at Austin*] I never heard about it in college either. So, whatever I know about prime numbers, I have learned from the newspaper and from hearing it mentioned occasionally by people. Some of my former students will remember that I used to say that there were not a "Great Eighteen Chorales" of Bach, but seventeen chorales. And when I figured that out, on the basis of the manuscript [*P 271, Berlin State Library*], which is very clear: there are 17 chorales, and then there's another one way over here. And it was the Bach-Gesellschaft edition that incorporated that [BWV 668] and called it "The Great Eighteen." But the chorale prelude that our artist performed last night [*O Lamm Gottes, unschuldig*, BWV 656] was part of what I call "The Great Seventeen." And that gets a certain amount of consternation from people, which is deliberate on my part. It's not a thing about it.

Phases in the creative process . . .

... for a writer

- conception / initial plan
- structure / content / words / editing / publishing
- buy and read the book or poem or journal article

... for an architect

- conception / site / funding
- program / detailed plans / working drawings / bidding / construction
- occupancy and utilization

... for a composer

- conception / commission / "inspiration"
- layout / detailed planning / writing the actual notes / revision / editing and printing / practice or rehearsal
- performance and hearing

[slide 2] What is the creative process for a writer? What is the creative process for an architect, for a composer? For a writer, she or he figures out the plan and 'I'm going to write a novel, or I'm going to write a poem.' 'What's going to happen here?' For the architect, she or he has to think about 'what am I going to build?' 'What am I going to plan?' 'How is it going to be paid for?' 'Where is it going to go?' What's the musician going to do? You have to figure out what you're going to write. 'Am I writing a symphony?' 'Am I writing a ballade for piano?' – 'what am I doing?' It might be a commission. 'Please write a euphonium sonata for me.' Or there might be just "inspiration." That's in quote marks, because it's not really true. That's not how it works. It's not really inspiration. [Technical problem: I'm sorry. That's what happens? I've just violated the 11th commandment button. I don't know if I can ... go back. I'm sure I pressed the wrong button. And that's what you do.]

So, the writer is doing structure and content and words. That's the creative process for her or him. [The] architect is having a building program, looks at the detailed plan and works it out, does the bidding, and they eventually get to the construction. What does the composer do? She or he lays it out, figures out what the eventual shape of this piece will be, then you plan it in detail. And you may write some actual notes down. You may invent some themes. You may figure out, in short, what

the music really is. And then, of course, you have to start revising. If you're smart, you don't take your first version; [but] keep working on it and then you edit it, and you print it and you practice or rehearse it.

For the writer, you buy the book. For the architect, we all go in the building, just as we came in this building today. For the musician, it's the performance and the hearing.

Well, what's that story? Why am I on that particular tack? Because what we have -- and this is the reveal -- what we have is something that Western composers have been doing for six centuries, unannounced, because nobody has talked about it. And there's only one figure in music history that talked about it. And if I can remember to tell you about it. I'll tell you who he was. Except for that, nobody's discovered this. So, I'm personally sort of intrigued by that, the originality of it. And the reveal is that composers -- when they began to lay out, lay out their music -- **they used the sequence of prime numbers for placing the salient moments in their compositions.** If I am writing a fugue, I would begin with a subject. The second subject might happen in a prime-numbered measure. The second -- the third entry of the subject could happen in other prime-numbered measures.

And this process of using the sequence of primes as a template is found beginning in early Renaissance music. Continues through the later Renaissance. It's absolutely clear in early Baroque music. Bach is filled with it, as we will see. Continues with Beethoven. If you look at the Third Symphony, the Fifth Symphony, the Ninth Symphony, and see where the fugue entries happen in the Ninth Symphony, they happen in prime-numbered measures. Nobody has seen this. Keep on going, and look at the symphonies of Brahms. The Fourth Symphony is rife with salient moments, thematic entries, fugal entries, climaxes, great moments that happen in [bar] numbers that are primes. His early work, the *Deutsches Requiem*, the German Requiem, is a dictionary of music that is organized by primes. If I had a score, I would show you that he does, in that first page, an entry in bar 3, an entry in bar 5, an entry in bar 7. He waits a long time. He gets to bar 11. He gets to 13. He makes the false entry at the 15th bar. It's not really an entry, and the choral part begins its thematic material at bar 17. So clearly, this is not a coincidence in my view, and I'm not making it up. Happens.

Number-Letter coding systems

A = 1	B = 2	C = 3	D = 4	E = 5	F = 6	G = 7	H = 8
I + J = 9	K = 10	L = 11	M = 12	N = 13	O = 14	P = 15	Q = 16
R = 17	S = 18	T = 19	U + V = 20	W = 21	X = 22	Y = 23	Z = 24



[slide 19] I was talking about the games that people play, and here's another one, the one on the left. And those are the numbers and letters game. B, what's the number? [two] A? [one] C? [three] H? A little slower there. H? [eight] And what is the sum of two and one and three and eight? [fourteen] Fourteen. Thank you very much. If you take the palindrome of 14, what is it? [waits for answer] Palindrome of 14 is 41. Palindrome is when you go backwards.

And 41 is a prime number. So, all of the things that I've just said in a somewhat of a jumble are salient to my thesis, and my -- I believe, discovery -- and the point of this afternoon's lecture.

Bach's use of number symbolism - I



Haßmann portrait, revised 1748

2✓ buttons on left sleeve

3✓ buttons on right cuff

5✓ buttons on coat

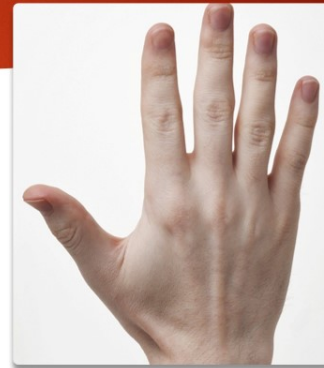
7✓ buttons on vest



B - A - C - H

2✓ 1✓ 3✓ 8

$$2✓ + 1✓ + 3✓ + 8 = 14$$

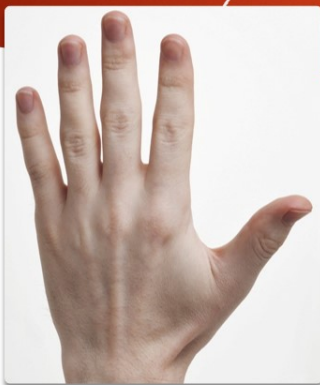


$$1✓ + 4 = 5 ✓$$

5✓ fingers

[slide 20] Bach used number symbolism in a number of ways, and I'm going a little quicker now. In the earlier portrait that we've seen a lot of, you count the buttons on his sleeve. There are two. On the right cuff, there are three. We count the buttons on the coat, and we count the buttons on the vest. And sure enough, they are all prime numbers. We've already done the panel in the middle, because that's the one you worked out for me so nicely. What about on the right? Can you think of any way that Bach would use the number 5 as a symbol of himself as a keyboard player? [Bodine holds up both hands with fingers extended] Can you? Of course. And what about 41? [reverses one hand] There is the symbolism that Bach used. He would do something special in the bar 14, and he would do something special in the bar 41, noting that, "Ah, yes, that's a prime number."

Bach's use of number symbolism - II



$$4 + 1 \checkmark = 5 \checkmark$$

$$41 \checkmark \leftarrow \rightarrow 14$$



J S B A C H

9 18 2 1 3 8

$$9 + 18 + 2 + 1 + 3 + 8 = 41 \checkmark$$



Haußmann portrait of 1749-50
(following Bach's stroke)

now 7✓ buttons on coat

now 14 (= B-A-C-H) buttons on vest
(note black drape over right arm)

[slide 21] The portrait was revised slightly. And then he had a stroke. Did you see what happened to his face? We'll go back one [slide]. Look at his face. Now look at the space. It's not clear in the portrait, or it's not clear in the screen, but if you look closely, extremely closely, [checks laser pointer: does this work today? No, it doesn't show anywhere.] There is on his right shoulder a black cloak, and it's very clear that the artist was indicating his illness through that black cloak in the portrait.

This -- all of it -- came to me really in the year 2013. I had finished my work at the university, graduated all the students, retired, did dinner, moved to Sanibel Island, had four wonderful years as organist of an Episcopal church there. The rector changed, so the musician changed. [quiet laughter] We came back to Gainesville, and I apparently was thinking about the "Musical Offering" of Bach and sure enough, somehow it clicked.

Ein Musikalisches Opfer, BWV 1079

- ▶ Meeting with Frederick II on May 7, 1747
(the 7th day of the 5th month in 1,747)
- ▶ Dedication printed on July 7, 1747
(the 7th day of the 7th month in 1,747)
- ▶ 1✓ and 7✓ and 17✓ and 47✓ and 1,747✓

a collection of chamber music items, including

- ▶ 2✓ ricercars (keyboard fugues)
- ▶ 5✓ numbered canons, all on one page, plus
- ▶ 5✓ unnumbered canons scattered through copy
- ▶ 1✓ instrumental sonata
- ▶ 13✓ pieces in total

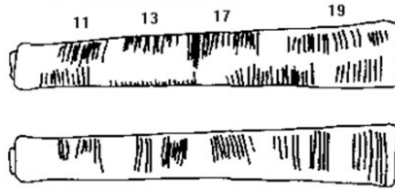
- ▶ R I C E R C A R
- ▶ 17✓ [9] 3✓ 5✓ 17✓ 3✓ 1✓ 17✓

[slide 3] And as you see, there are two ricercars in that work. There are five numbered canons on a very straight page, and then there are five more canons scattered through the whole Musical Offering. So those are both five and five. There's one sonata, and the total of that is 13, which happens to be a prime number. It looks like we're getting close to proving our point. We can go a little further, Bach. To visit his son, second son, Carl Phillip Emanuel, who's the harpsichordist for Frederick II of Prussia, we call him Frederick the Great. I guess the Germans say *Friedrich der Große*, so that's, that's right. At any rate, he managed -- Bach did -- to get there in the fifth month on the seventh day of 1747, and all of those are prime numbers, one thousand seven hundred forty-seven. I bet you didn't remember that. That's a prime number. Well, there's a section of the talk that takes eight minutes, and I can't do it right now. If there happens to be time, come back. It's a discussion about how the "Musical Offering" works. At any rate, he wrote it in two months, which was amazing. Put it together, published it, and surprisingly wrote the entire exact date of publication. If you look through the works of Bach and of most composers, they put the year. Maybe they put the month. He wrote the day, which turned out to be, of course, a prime number. Turns out to be [if I push the button] the seventh day of the seventh month, and since it was only two months' worth, is still 1747. Okay, we'll have to save that eight-minute little segment for another day.

Ah, *ricercar*. The name. People have wondered why Bach used the word *ricercar* instead of just saying "Fugue." If you go back and look at the number code, it makes a mess. But if you look at the number code of *ricercar*, it happens to fit. They are all prime numbers except for the "I". So, clearly, and I believe this is past coincidence since it's everything you look at, Bach is paying attention to the sequence of prime numbers as he assembles and revises his "Art of Fugue." [WRB meant to say "Musical Offering" here.]

Various sequences of numbers

- 1s
- 2s
- 5s
- 10s
- 12s
- 1 - 2 - 3 - 5 - 7 - 11 - 13 - 17 - 19 - 23 - 29 - 31 - 37 - 41 - 43 - 47 - 53 - 59 - 61 - 67 - 71 - 73 - 79 - 83 - 97 - 101 - 103 - 107 - 109 - 113
- Sieve of Eratosthenes, ca. 200 BCE (SEE ANIMATION IN TITLE SLIDE)
 - except for the number 2, all primes are odd numbers
 - after the first decade, all prime numbers end with either 1 or 3 or 7 or 9
 - Throughout history (until the 1977 invention of public-key cryptography), prime numbers had no practical value or use (except to mathematicians)



[slide 4] This, we've already done. We did this at the beginning. We did the sequences of 1s and 2s and 5s and 10s and 12s, and you did that very well. There [points] are those primes coming up. As you came in, a few of you puzzled out the tease on the board [on the screen] which was that grid with blinking lights. And that is an animation of the Sieve of Eratosthenes, which is a way of finding prime numbers. And if we could take the time to go back, we can. You'll discover that that is a simple way. It lasts as long as your patience lasts. To find prime numbers. [technical problem: I did it again. Let me see. Why? That's twice. That's also the punch line of a joke that I can't tell. There it is. And there we go.]

[moves through slide] One of the things that is interesting about prime numbers is that, except for the number 2, they are all odd. It's also easy to remember that after you get past the first decade (which has more exceptions than it has rules). and you get to the 11, they all end with either 1 or 3 or 7 or 9. So you can count on a prime number ending with 1 or 3 or 7 or 9. Period.

That's what I just said. [tests pointer: It's working.]

[moves] And this is interesting, and which sort of gives me a thrill, that prime numbers were absolutely useless. Nobody could figure out anything to do with prime numbers except the mathematicians. And what they did was, in their silo, think about prime numbers. Happens to be the fundament of mathematics. So, it's very important to them, but it wasn't important to anybody else. They [prime numbers, not mathematicians] were useless until in the third quarter of the 20th century. With public key cryptography, it became possible to use very high prime numbers as part of making your bank account secret and your credit card transactions secret.

So, the fact that Western composers for six centuries have been using this sequence of prime numbers in their compositions is something that sits over in our silo. That's over in our music theory silo. Here in this building [points to Music Building]. And over in Walker Hall, I think it is, there are the mathematicians, thinking about their prime numbers. And twain have not yet met. We're working on it.

J. S. Bach, Fugue in C minor [Passacaglia], BWV 582b

EXPOSITION 1	EXPOSITION 3	FINAL SECTION
five-entry exposition -- with pedals	five-entry exposition -- with pedals	(= episode and coda, with special events)
1✓ S tonic A	53✓ S dom	103✓ S tonic final subject entry
5✓ S dom S	65** S tonic -2 [67✓]	113✓ harmonic poise on dominant
13✓ S tonic B	77** S dom -2 [79✓]	117 N ⁶ cadenza moment = hemiola
	S	
17✓ S dom T	87** subdom -2 [89✓]	119 display of entire range of pedal clavier
23✓ S tonic A		121-24 crescendo of added voices
EXPOSITION 2	(** These three subject entries would also have begun in prime measures, if the bar 57-65 episode had been 2 bars longer and the bar 91-103 episode 2 bars shorter)	
two-entry exposition -- manuals only		
29✓ S rel major		
41✓ S dom of rel major		

[slide 5] We're down to examples. I have five examples. And they're drawn from the organ literature, since we are doing organ music this week. But I could do, as I think I said, I could do the same lecture on the choral literature, starting in the early Renaissance and going to the present day. I could do the same lecture starting with Bach's Brandenburg concertos and going through the orchestral literature through Bach, Beethoven, Brahms, and so forth. Could do the same lecture with any musical area. It works. Because this process of using this technique, which was rather secret, apparently, because nobody talked about it. I can't explain that yet. I'm working on it. But I have no idea why nobody spoke of this.

The single exception to what I just said is a theorist and composer. Are there any woodwind players here? [hands come up] Okay, Antonin Reicha, important woodwind composer, professor of composition of Paris Conservatory. He had the best flute player, the best oboe player, the best French horn player in Paris, in France. He could write anything, and that he did. And so, the woodwind quintets of Reicha are the foundation of the woodwind quintet literature.

He also wrote [treatises] extensively, and at one point, actually at several points, he said something like (and I'm paraphrasing his French) "Prime numbers would be an interesting way to build phrases." In another place, he said, "When we structure music, prime numbers would give us an alternative to the regular four bars, four bars, four bars." He is the only person -- and I have searched, as you can tell -- he's the only person who's ever referred to this. It reminds me of the secret of bell tuning, for which there are some carillonneurs here. You know, it [bell tuning] was well understood in the 16th, 17th centuries in Holland. And they kept it so secret that they lost it. And so, the secret of bell tuning was lost. Nineteenth-century bells are uniformly bad because they didn't know how to tune them. They had lost the secret until an Englishman figured it out in a different way by doing the tuning fork system and so forth. And that was a little sidebar, and I've consumed a few more moments of my time.

[pointing to slide 5] You're looking at a dissection of the C minor fugue [BWV 582b], the one that goes with the Passacaglia. And the key to finding the [prime] numbers in it is to number the [bars of the] fugue separately. And nobody had done that -- the editions certainly don't. But if you start with [bar] number 1 at the point where the fugue subject begins, you discover what you see in the chart, that the first, [technical problem: my pointer doesn't work.] So, you can tell, left-hand red column, the first subject is in bar 1 of the fugue, measure numbers of the fugue, The next one's in bar 5 and the next one in bar 13 and the next one in 17, 23. That's pretty much beyond coincidence. In the middle section of exposition 2 [I'm lost in the chart: I've skipped it.] It's at the bottom left. There they are, in bar 29, 41. How would you expect to find a fugue subject in something like bar 29 and bar 41, unless you were understanding exactly the sequence of primes? It was on Bach's mind. In the middle section, the manual section of the fugue – [I'm going to be pardoned, I've misspoken.] In the third exposition, the first entry is in a prime measure, and the next three are moved. They are displaced by two bars. And in statistics, you call that a deviation. It has a 'thing' of deviated that has to do with the length of the episodes on either side. So apparently in the revision, I believe, that Bach had originally had those entries in prime numbers; and then he played with the episodes and moved that group two bars away, deviating. And then we get to the final section. We're still having prime-numbered events, and prime-numbered events. So that's the C minor Passacaglia.

J. S. Bach, Fugue in E minor, BWV 548b ("Wedge")

Vivalsi-concerto form with ritornelli and concertino-episodes

(-) prime bar	(+) entry or event	(-) prime bar	(+) entry or event
1✓	Subject tenor tonic	89✓-97✓	bar 93, concertino-episode
5✓	Subject alto dominant	97✓	concertino-episode
13✓	Subject soprano tonic	107✓	Subject soprano relative major
19✓	Subject pedals dominant	113✓	bar 112, concertino-episode
23✓	concertino-episode	137✓	Subject pedals supertonic
31✓	2 bar 33, Subject alto tonic	157✓	bar 156, Subject alto submediant
43✓-47✓	bar 45, Subject tenor subdominant	173✓	recap of opening ritornello begins
1	53✓	1	179✓
	bar 52, Subject soprano tonic		bar 178, Subject alto dominant
	59✓		181✓-191✓
	beginning of manuals concertino-episode		bar 186, Subject soprano tonic
	67✓		191✓
	1 ending of concertino-episode		Subject pedals dominant
	67✓-71✓		199✓-211✓
	bar 69, Subject pedals tonic		bar 206, Subject alto tonic
	79✓-83✓		211✓-223✓
	bar 81, Subject pedals dominant		bar 217, Subject tenor subdominant
	89✓		Subject soprano tonic
	Subject alto subtonic		223✓

5✓ big pedal entries

[slide 6] I'm now into my rapid discussion of organ literature. And there's the next one. This entire presentation should have been given -- I didn't say this to you -- should have been given in 2017, when Patricia and Peter Dewitt organized an A.G.O. conference in Jacksonville, and I was on the bill. And I had to withdraw, and they were sorry. I was very sorry. But I have knowledge now, not only that they knew this before somebody knew it, but that he [Peter Dewitt] has a stunning performance of the "Wedge" fugue on the internet if you can find it.

What Pete doesn't know is that I have the "Wedge" fugue here. Can you see that? [points at paper strip spread on the floor] Take a look at it. It happens to be the "Wedge" fugue in open score with each prime-measure event numbered. Just out of curiosity, can you imagine in 200 years when they

excavate this building? They're going to find a roll of paper 18 feet long, it is an open score, it's an organ fugue, they'll have red marks every so often, Can you imagine the archaeologists trying to figure that out, can you imagine?

Johannes Brahms, Fugue in A-flat minor, WoO 8

First version, 1856: <small>(23✓ entries of subject plus four entries in diminution)</small>	bars 1✓ - 8	bars 9 - 10 <small>(deleted)</small>	bars 11✓ - 16 <small>[episode]</small>	bars 17✓ - 22	bars 23✓ - 24 <small>[episode]</small>		bars 25 - 32	bars 33 - 41✓	bars 42 - 47✓
Soprano	S ^{inv}			S ^{dir} S ^{inv}		S ^{inv}		S ^{dir} S ^{dir/sync}	
Alto	S ^{inv}			S ^{dir}	S ^{dir}	S ^{dir}	S ^{dir}	S ^{inv}	
Tenor	S ^{dir}	S ^{dir}		S ^{dir} S ^{inv} S ^{inv}		S ^{dir}	S ^{dir} S ^{dir}	S ^{dir}	S ^{inv}
Bass	S ^{dir}	S ^{dir}				S ^{dir}	S ^{inv}	S ^{inv} S ^{inv/aug}	S ^{dir}
Published in 1864: <small>(29✓ entries of subject plus four entries in diminution)</small>	bars 1✓ - 8	bars 9 - 16 <small>(new)</small>	bars 17✓ - 20	bars 21 - 29✓	bars 30 - 31✓	bars 32 - 37✓ <small>(new)</small>	bars 38 - 43✓	bars 44 - 52	bars 53✓ - 58

[slide 7] Brahms. What a figure. Never married. Had several women that he was strongly interested in. Composed in private in the mornings and went for walks in the afternoon. That sounds like a pretty good way to go at it. Apparently, in his quiet mornings he counted prime numbers in this compositional process. And I've already said about the German Requiem, how that in every movement, it's a textbook example. This is a little fugue in A-flat minor. What is the relative major of A-flat minor? [waits for audience answer] I'm not going to tell you. C-flat's it -- C-flat, I heard it from the gallery over here, thank you. C-flat. How many flats does C-flat have? - Seven. - I wouldn't say it's a prime number, that's not one. But that's a lot of flats.

Brahms was very sad because Robert Schumann was dying. And so around the time of his birthday, Robert's birthday, he [Brahms] wrote a fugue dedicated to Clara, who was his crush. And it is this fugue in A-flat minor. The first version, which is the early manuscript, it's the top panel. Let me see if my explanation tells you anything. [Again, I'm sorry for the pointer.] The left, the yellow strip, gives the sections of the piece according to their bar numbers. The first section is bars 1 through 8, and we skip. The next section of the fugue is bars 11 through 16. The next section is bars 17 through 22. The next section is bars 23 through 24, we skip the green, the next one is bar 35, 35, 32, skip, next to last is 33, 41 and the last one is 42 and it has 47 measures in that fugue.

A few years later, what was it, eight years? Do your math on us? Yes. He revised it and published it. What he did in that revision must have taken some skill. For those of you who study 18th-century counterpoint, you'll appreciate it if you have a complete fugue and you decide to put in a section here and a section there. And like the whole thing still coaching, well that's what Brahms did. And so, the two green columns are Brahms' addition and the second part, the lower yellow strip, has the bar numbers for the published version, which is the one we play. And if you look closely, they're red, which helps. You see that Brahms is still paying attention to the prime numbers in the second version. And that cannot be coincidence. There is no way that that's just gonna happen.

I wish we could spend an afternoon just on this fugue, and this part was huge. It's also hard to play. There is an edition on the market that transposes it up into A minor, which is a lot easier to read. [laughter] If you find an organ that plays a half-step low and you could take that copy to that organ and hear what you're supposed to do. That's too much. And interestingly enough, [Oh, I went back. You see what happened to me? I must have pushed that in enjoyment. They're excited. I'm Brahms -- I'm just pointing out something.

[goes back a slide] Go back to the "Wedge" for half a second. There's the "Wedge." And there are five of the pedal entries. Five is a prime number. [laughter] Thank you.

So, Brahms did not make his published version come out to 59 bars. What's your point again? If I were Brahms, I would have added a bar right there just to make it a bit more, but he knew better. So that's two examples, three examples. I've done Passacaglia, the C minor fugue only. I've done the "Wedge" – which, Pete, is still on the floor. You get excited about the music that is spread out on the floor. And Brahms.

César Franck, Choral No. 3 in A minor

The image shows a musical score for César Franck's Choral No. 3 in A minor. The score is written for piano and features two staves. The tempo is marked 'Largamente' at measure 15 and 'Più largamente' at measure 19. Two vertical pink lines are drawn over the score, with boxes containing the numbers 17 and 19, indicating specific points of interest. The score includes various musical notations such as notes, rests, and dynamic markings.

[slide 8] César Franck. I've tried to present each of these examples in a different way, and this way is the actual score itself. And I've marked a big slash, and in a box in red pen, I have marked the bar number, because the Durand edition is not numbered, and so I had to mark them, had to count. So, we see that in the, [points] it's about the third section of the opening, suddenly Franck starts with these big crashing chords, and the second one happens in a prime and the piece that continues even the third time in bar 19. The same thing happens at this point in the piece. How many of you know this piece well enough to sort of stay located in this? I'm apologizing to the rest of you. I'm taking it from me. I'm taking pieces out of the piece. Just the way I'm presenting this.

Ajoutez Jeux de fonds de 16 p ôtez Anches G O

Rit.

dim

Otez Anches Positif

Otez Jeux de fonds de 16 p.

RECIT mf

[slide 9] The chorale, which is the reason it's called *Choral*, is beginning right here [points at bar 31] – which, fortunately for my lecture this afternoon, begins in a prime-numbered measure, which is always nice. Point to what is true!

46

47

pp

POSITIF

p

4

53

58

RECIT SWELL

p

[slide 10] When the chorale first statement ends, it's bar 47. That's [points] the end of the chorale, and we get back to more toccata, we get the crashing chords in bar 58, and we get some more toccata, and then at the bottom of the page,

79

POSITIF

Franck - Choral No. 3 in A Minor

97

RECIT. Jeux de fonds 8 p. Hautb Tromp

Adagio

POSITIF. Flûte et Bourdon 8 p

dolce espress

cresc.

PEDALE Jeux doux

this [slide 11, points at bar 97] is the beginning of the middle section, the lyric part, the part that you play for a prelude, if that's all we've got. That begins in a prime-numbered measure.

I don't mean to sound redundant, but Frank was paying attention to the sequence of prime numbers, in my view, as he proceeded in the piece.

Otez Tromp du Recit

129 a Tempo

RECIT Ajoutez q q jeux de fonds de 8 au Positif

131

G O Jeux de fonds de 8 p Claviers accouplés

RECIT

POS

PED. morns douce

157

156

This [points to slide 12] is top score in the place where the left hand moves to a bigger foundation after the whole lyric middle section is done. Then bar 157, that's the reference to the chorale theme that happens to happen over the toccata figures.

Handwritten musical score for organ, measures 171-190. The score is for "Anches positif" and "Anches Pédale". Measure 173 is highlighted with a red box and a red line. Measure 191 is also highlighted with a red box and a red line. The tempo marking "Rit." is present above measure 191.

And this [points] is the final statement of the – [I beg your pardon. It's not toward the end. What's going on? Oh, there's a difference.] This [points to slide 13] is not a continuation. So, [bar] 173 is the beginning of the final statement of the chorale over the toccata figures. And at the bottom, you find you're near the end of the piece. So, it's quite clear. Let's go back.

So it's quite clear that in putting together this work, which was his final work, Franck was actually in the last few days of his life up in the organ loft of Ste. Clothilde, working on registration. And one of the students had to complete a few details because he never wrote them down before he went to the publisher.

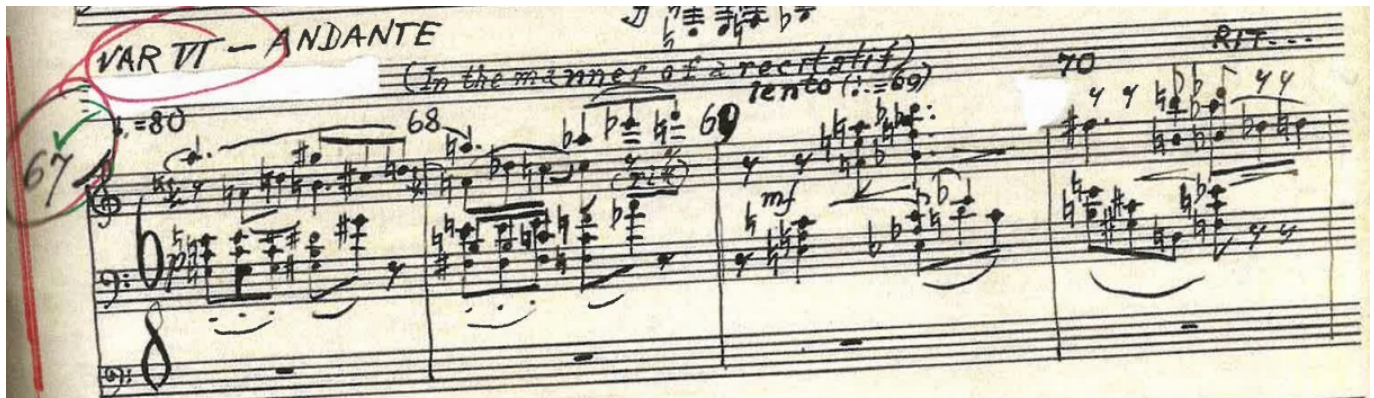
Arnold Schönberg, Variations on a Recitative, Op. 40

Handwritten musical score for Arnold Schönberg's Variations on a Recitative, Op. 40. The score is for piano. It includes markings such as "8 poco a poco ritg", "dim", "10", "11", "12 VAR. 1", and "64". The tempo marking "a tempo" is circled in green. The score is written in a complex, modern style with many accidentals and dynamic markings.

[slide 14] My final example is a work that some of you may have run into 30 or 40 years ago. Arnold Schoenberg was a big name in music. I think among composers he is less of a big name now. In my view, the “serial music” approach has begun to fade, if not completely faded, but this is an organ work of Schoenberg who was a big name. I cannot tell you if the work is a serial composition; I just don't happen to know. But this [points] is Schoenberg's manuscript, and for some reason, he has numbered the bars carefully. This is probably - this part - is just his way of working, in making a fair copy after he had completed the composition. He numbered the bars carefully. Well, I went through and I looked at every prime-numbered measure in the manuscript.



And sure enough, I kept finding that at important moments in the piece -- this [points to slide 15] is the beginning of the second variation.



This [slide 16] is a pedal entry of what is thematic at the beginning of the sixth variation.



slide 17] Beginning of the seventh variation is bar 89, and this toccata material at bar 113. Now the composers and organists both will be interested, but Schoenberg was notating the 16-foot part at [actual] pitch throughout this work. It's a little bizarre because organists usually know you don't have to do that, and certainly orchestral conductors and writers know you don't have to write the contrabass part an octave lower – it's down there, but you don't have to do that. But throughout the

work you just have to know what you are dealing with. It looks like there are some notes that are off the horn somewhere down there, but he's just notating the 16' pitch. The H. W. Gray edition of the Schoenberg "Variations" maintains that. And I personally think that's a little confusing, but that wasn't something they asked me about.



[slide 18] The end of the work? Close to the end-- that's not near to the end, but it's close. At any rate, I think I've made my point with the five examples. Of the five examples, that Bach, that Bach again, that Brahms, that Franck, and that Schoenberg, each over that range of the organ literature, paid close attention to prime numbers as they were laying out their compositions, since they were seeing where things happened.

So, for once, I have not run over. I'm going to ask where we are in time because I don't know. [Dr Ellis: Perfect. You're perfect in 3:40, so perfect time to add any questions.] [WRB continues] Okay. Perfect. I think I've made my point. I could go on. I could give you the whole "Musical Offering" story. But I think you're getting the point. This is, for me, an ongoing matter. I would love to have a graduate seminar of graduate students who would sit down and do all the work. [laughter] I would love to be writing a dissertation at age 26. None of that's going to happen but it's original enough to fit into that category.

So let me ask if you have questions about what I've talked about and if I can possibly answer them. [audience member] I actually have two questions in one. Is the concept of prime developed by the German school? [WRB] I'm not understanding. [audience member] Is the concept of prime developed by the German school of writing music? And the next question is, is this concept of prime somehow having tested through human DNA?

[WRB] I don't know. I'm not able to answer that question. As far as I can tell, the prime numbers concept in mathematics is in its own place. And the musicians that are using prime numbers are in their own place. And I have not found any connection. Am I answering your question? [audience

member] Because if you listen to the music-- I mean, Bach is the beginning, the middle, the end of everything that's part of that. But when I listen to Bach, he almost catches us in a special way, that we humans feel attracted to that music. Perhaps there's something in our sequence of DNA that makes prime *[numbers]* make sense. *[WRB]* This is above my pay grade, to say whether prime numbers are something visceral, whether they are something natural that comes out of the body or through the universe or in those places. I have no idea and, as I say, that's above the pay grade. I can't give you that.

Yes sir. *[audience member]* Does any of this possibly have to do with Egyptian, Masonic, philosophy? *[WRB]* The music of the universe kind of story. It's very possible. I have not found any use of prime numbers in studies of science in the early modern era. I haven't. As far as I'm aware, the prime-number theory and discussion and awareness has simply sat in the mathematics silo until the third quarter of the 20th century. I don't think anyone else has paid that attention, to me as far as I know. Question? *[audience member]* It would be fascinating to be able to speak with them *[question garbled]* that would just feel where it should be. – *[WRB]* Yeah, it would be fascinating. Hey, hey, Joe, that's John, I guess, Sebastian. How did you feel about that prime number? Did you do that, did you mean to do that? Johannes, Joe, another John. Could I, could I ask you what you were doing that morning when you were writing the Requiem? I say again, I don't know. I can't -- I can't really say that. I have a suspicion that they were deliberate because it's hard for me to imagine that this could have simply happened to a composer. It's hard for me to imagine . . . but oh, we're getting into the serious stuff. You see what you missed by not scheduling me for two hours? *[laughter]*

using prime numbers as templates -- #1

Renaissance masters

- 1527 – Adriano Willaert, *In convertendo Domine* (Psalm 125/126)
- 1564 – Orlando di Lasso, *Bonjour, mon coeur*
- 1566 – Alessandro Striggio, *Che farà fede al cielo*
- 1570 – Thomas Tallis, *Spem in alium*
40-voice motet for eight 5-voice choirs
(Mary, Queen of Scots = M-A-R-I-A =
= 12 + 1✓ + 17✓ + 1✓ + 11✓ = 40)

English virginalists and Sweelinck

- 1602 – Peter Phillips, transcription of di Lasso, *Bonjour, mon coeur*
- 1602 – Peter Phillips, transcription of Striggio, *Che farà fede al cielo*
- 1612 – Orlando Gibbons, *This is the record of John*
- 16-19✓ and 1,619✓ –
Jan Pieterzoon Sweelinck,
“Hodie Christus natus est”
(#13✓ of *Cantiones sacrae*)

North German School

- 1648 – Heinrich Schütz, *Geistliche Chormusik*, Op. 11✓ (29✓ motets)
- 1670 – Dieterich Buxtehude, *Wacht! Euch zum Streit* BuxWV 100
(lengthy cantata)
- Eins bitte ich vom Herrn*
(Psalm 27:4), BuxWV 24
- Preludes and Fugues for organ

[slide 27] Here we're starting a list of compositions.

using prime numbers as templates -- #2

Bach and his circle

1707 – Passacaglia and Fugue in C minor
1707 – Cantata 4, *Christ lag in Todesbanden*
1723 – Motet III, *Jesu, meine Freude*
1724 – *Johannis-Passion*, BWV 245
1727/29 – *Matthäus-Passion*, BWV 244
1731 – *Clavier-Übung I*, Six Partitas
1747 – *Ein Musikalisches Opfer*, BWV 1079
1743-50s – *Kunst der Fuga*, BWV 1080

Wilhelm Friedemann Bach (1750s)

Johann Gottlieb Goldberg (1750s)

Mozart and Beethoven

1779 – Leopold Mozart, *Missa Solemnis* in C
1783 – Johann Georg Albrechtsberger, *Doppelfuge für Orgel*
1779 – Wolfgang Amadeus Mozart, *Vesperae de Dominica*, K. 321
Requiem, K. 626
1806 – Michael Haydn, *Requiem* in B-flat
1819 – Ludwig van Beethoven, *Missa Solemnis* “Credo” and “Sanctus” fugues
1819 – Piano Sonata #29✓ in B-flat, Op.106
1821 – Piano Sonata #31✓ in A-flat, Op.110
1824 – Symphony No. 9 in D minor (“Choral”)
1826 – Quartet, Op. 133 (= *Grosse Fuge*)

Berlioz, Brahms and Wagner

1830 – Hector Berlioz,
Sinfonie Fantastique, H.48
1856 – Richard Wagner, *Tristan und Isolde*,
Prelude to Act One and “Liebestod”
1862 – Johannes Brahms,
Ein deutsches Requiem, Op. 45
1884 – Symphony No. 4, Op 98

[slide 28] In mixed company, I cannot discuss the Prelude to *Tristan und Isolde* in actual terms. I can tell you, however, that the prime numbers are important at the moment when the Tristan chord happens. And again, I have to save that for another time.

why? why? why? why? why? why? why?

- These appearances of prime numbers are a complete coincidence . . . OR
 - They just happen “naturally” . . . OR
 - There is some cosmic significance . . . OR
 - The entire matter is a result of apophenia, selection bias, or confirmation bias . . . OR
 - There is some as-yet-undiscovered pattern in the sequence of primes . . . OR
 - Primes were self-evident in intellectual/musical circles of earlier times . . . OR
 - The primes technique was absorbed by creative people by observation or experience . . . OR
 - It was a deliberate design technique, passed from teacher to pupil (but then was carefully kept secret, in the ages before copyright) . . . OR
 - It is a design procedure discovered anew in each generation

[slide 31 + dialogue paraphrasing the slide points] Oh, my [laughter] . . . this is a complete coincidence. That's what some of you were fishing for. But it just happened, you know -- our stomach turns and there's a prime number. Or they're cosmic [inaudible] and then the cosmic. And then the psychologists try to tell you why it happened. Nobody in movement, the mathematicians has found a pattern in prime numbers. It just happened.

Yes, that's a question. [audience member] I just wanted to know if you have looked anything into, like, the holy numbers, maybe like a Christian world, you know, we think of like 3 and 7, which are both prime numbers. [WRB] They are. [audience member] I kind of wonder if that might be true. [WRB] And 12 is one of them, and it's not random. That sort of short circuits the idea. As soon as you begin to carry a sequence, like the holy numbers, the Biblical numbers, as soon as you start to carry it forward and discover that you got a coincidence at three and seven and it falls apart. That can happen in a piece of music. You find something happening in bar 3 and in bar 7 and bar 9, whoops, bar 12, well that could be one off, bar 15 -- the bar 3 and 7 meant nothing because that was coincidental, it had nothing to do with the sequence.

Is that a question yet? [audience member] Or this like of the significance of the primal sequence and your experience of researching all this music, does music that makes use of the primal sequence, does it in your opinion like sound better or is it more satisfying to listen to, compared to pieces that really don't make use of the primal sequence? [WRB] I have two parts to the answer because I do have thoughts about that. One is that this makes no difference to performance. I really think this has nothing to do with performance. It's why I was a little shaky on asking to do this event at this [conference] because we're performers and I don't think it [primes] really has to do with performing. It has to do with composition. So, I belong in the theory department when I'm doing this, not the performance department. And the other part of the question, if I can recall, I enjoyed that first answer so much. [laughter]

[audience member] I guess, like, do you think that music that makes use of the praxis? [WRB] Oh, whether it makes the music sound better. Yeah. No, I don't. That's what I think. I don't think it makes any difference to the sound. I think it was simply a matter, I believe, that it was simply a matter of how the composer felt in trying to arrange things. If you are composing and you put all your pencils in a row, and you put the Scotch tape here, and you can't get started unless the notepad is here, and you have this obsessive-compulsive . . . that's part of how you feel, that has nothing to do with how good the music is. So, the fact that the composer followed this technique, I don't think really has to do with it. [audience member] Like a choreographer? [WRB] Yeah, it could be. It could well be. It reminds me of [the chord progression] VI II, V, 1, which we've heard in one of the pieces over there that did that. I can't remember. Getting off the track.

Yes. [audience member] Have you ever thought, I wonder, if this composer has followed the sequence and then looked at it and they absolutely didn't, like if you aren't into anything, they should have not done it. [WRB] Yes, yes, I haven't found it much in Handel. What I did with Handel was take the organ fugues and I went through the major choruses of *Messiah* and I could not find any evidence of it, so he doesn't seem to have paid attention to that. Mozart, by all my examples, I think I have one piece that he might have followed it, but it was probably a coincidence. So, yeah, it is. And for me, it is fairly clear, as I said, in the first decade, the first 10 measures (of a composition) you can't really tell much. But as soon as you get up to bar 19, and bar 23, and bar 29, and bar 31, you can start to tell if the composer is paying attention and is placing salient moments, fugal entries, fermatas, grand pauses, whatever. Changes of registration (we're organists.) You can tell that the composer is doing that, or is not.

We've got a time check. – [Dr Ellis: Yep, like one more question, I think, 'cause then we have to set up for the next person.] [WRB] Yep. – [audience member] Does all of Bach's music follow the same pattern? – [WRB] I can't tell you. The pieces that I have looked at, in large measure, do. The longer the piece, the more likely it is. I can give you a preview that I'll give, of a lecture that I'll give in a couple of months

on the week past Bach's birthday. So it will be on March 28th of this year. Yeah, March 28th. In this room, at one o'clock, as part of the School of Music Creative Lecture Series, and it's going to have to do with the completion of the last fugue of the "Art of Fugue", which Bach did not finish. He died. Made it difficult. *[laughter]* So, the fact is that when you look at the parts that he did do, that he finished in so-called Contrapunctus 14, every fugue subject entry happens in a prime measure. And so, in the completion that I have written, I kept on doing that.

[audience member] Are they part of the 14? *[WRB]* Yes, Yes, this was kind of deliberate. In fact, we're going to go back to symbolism and talk about the third section of Contrapunctus 14 and how the theme subject, B.A.C.H. That . . . I don't . . . you can't give me that time. That's what I have to tell you about.

[Dr. Ellis: Thank you!]
