15th Annual Southern Graduate Music Research Symposium, University of Florida
Keynote Address by Willis Bodine, UF Professor of Music, emeritus
January 17, 2026, 1:30 pm in MUB 101
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[John-Peter Ford begins] Good afternoon, and welcome to the afternoon sessions of the 15th Annual Southern Graduate Music Research Symposium. Just as a couple of reminders for those that are joining us now and missed the earlier announcements this morning: please, no food in the auditorium, and if you do bring a drink in here, we ask that it does have a top or a lid on it, and if you do spill something, please clean it up. There are napkins provided in the bathrooms. Other than that, *[reads]* I am pleased to bring you our session number three for the day: ***Primal Bach: Two Divergent Silos***. A native of Texas, Professor Willis Bodine was University Organist and Carillonneur at the University of Florida from 1959 to 2003; conducted the UF Chamber Singers from 1966 to 1981; traveled to Holland, Germany, and France under grants for musicological research; and was musical director of The Willis Bodine Chorale, 1986 to 2004, in Gainesville, Florida. His musicological study at The University of Texas at Austin, '57 and '60, was with Paul A. Pisk and Richard Hoppin, two mid-20th century luminaries. Under a Fulbright grant, he studied organ, harpsichord, and chamber music at the Nordwestdeutsche Musikakademie in Lippe-Detmold, Germany, from 1957 to '59. His former UF students now hold college and church positions worldwide. Universities and churches in North Carolina, Texas, Ohio, and Florida have commissioned Bodine's more recent choral and carillon compositions. With his wife, the singer and writer Anna Schoff Hartung, he has two children, two grandchildren, ten great-grandchildren, and one great-great-grandson. *[audience murmurs]* He is a member of the Association of Anglican Musicians, the Guild of Carillonneurs in North America, the Music Teachers National Association, and the American Guild of Organists. Please welcome Professor Willis Bodine. *[applause]*

2'30" *[WRB begins]* Thanks very much. If the volume is inadequate, please raise your hand. Otherwise, I'll speak at about this level for the 40 minutes that I intend to talk, and then I will look forward to your questions and any other things that you want to bring up.

2'50" Slide #1 *[title slide, showing as audience arrived]*

The ‘tease’ is an animated version of the Sieve of Eratosthenes, and it is a way of finding out what numbers are primes. And since I'm sure that you have figured it out already, I will only point *[points with laser pointer]* to the right-hand column, which is the list of primes. And there will be a test on that at the end of the hour. *[polite laughter]*

3’20” Slide #2

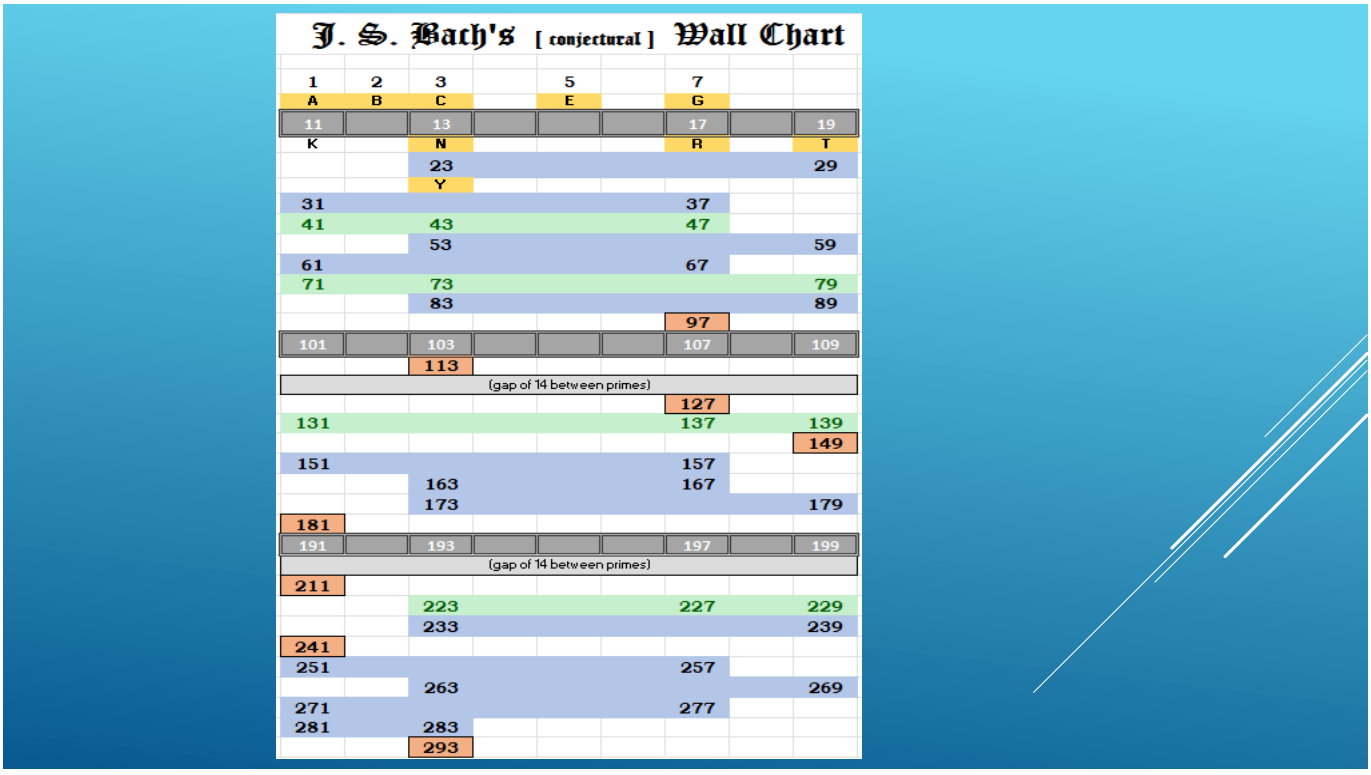
This is a picture of a modern American university. The silos are a metaphor for the areas of knowledge that tend to be sequestered, isolated, very much separated from each other. Those that look in the bottom of the silo of mathematics will eventually get out of the applied section and get down to the pure mathematics. And when they keep looking down there, they'll find number theory, and eventually they'll get to the point of prime numbers, which secretly is my topic. In a different silo are the musicians. And of course they're practicing. Those are the performance people -- that trombone player who's been practicing since 11 o'clock this morning with his door open *[murmured laughter].* And then you get a little lower in the silo, and you get down to the music education people and then the music literature people. And eventually you get down to the theory-comp folks, who are in their very carefully sequestered silo *[gestures]*, and the composition faculty -- which has to teach essentially one-on-one. There's not much you can do in a large class to teach composition, and so composition has tended to be taught one-on-one.

So, there *[points]* are the one-on-one folks. And over here *[points]* are the prime-number folks. And until now, ‘the twain have not met.’ The fact is that many Western composers have used the sequence of prime numbers as a template, as an outline for laying out their compositions. That has been true -- to my investigation -- for the last five or six centuries. My earliest piece in my list of examples is Machaut, and going on down to the present day. If Paul Richards were here, I would cite his orchestral work which quotes Beethoven [*Return the Echo,* 2019]. And I want to chat with him at some point and find out if he meant to use the prime numbers -- because they're there in a very clear sequence, and I'm not sure about it. We'll discuss that with Paul. I don't want to dwell on that.

1436	Guillame Dufay	Motet: *Nuper rosarum flores*
1515	Josquin des Prez	*Missa Pange lingua*, „Credo“ movement
1527	Adrian Willaert	*In convertendo Domine*, Psalm 126
1619	Jan Pieterzoon Sweelinck	Motet: *Hodie, Christus natus est*
1680	Dietrich Buxtehude	Cantata: *Membra Jesu nostri*
1749	Johann Sebastian Bach	*Kunst der Fuge*, Contrapunctus 14
1824	Ludwig van Beethoven	*Symphony No. 9 in D minor*, second movement
1830	Hector Berlioz	*Symphonie fantastique*, fifth movement
1869	Johannes Brahms	*Ein deutsches Requiem*, first movement
1922	Alban Berg	*Wozzeck*, Act One
1936	Béla Bartók	*Music for Strings, Percussion and Celesta*, first movement

6′08″ Slide #3

[points to screen] Here's my list. This list doesn't even go back to Machaut. And we're going to begin today with the example of Buxtehude. How many examples are there in this list? *[waits for audience answer]* Eleven. How many are there starting with Buxtehude and going down? *[waits]* Seven. Eleven is a prime number. Seven is a prime number. That's not a coincidence. Let's jump in.



6′45″ Slide #4

Whoops, we've got to do one more thing. And that is my conjectural mental wall chart, which J. S. Bach used. Bach kept in his mind the various prime numbers, and those are the ones in black *[points]* in these columns. You will notice several things about the chart, which was a mental chart. All of the prime numbers end in either one, or three, or seven, or nine. No exceptions. *[WRB chooses not to discuss the number 2 at this point.]*

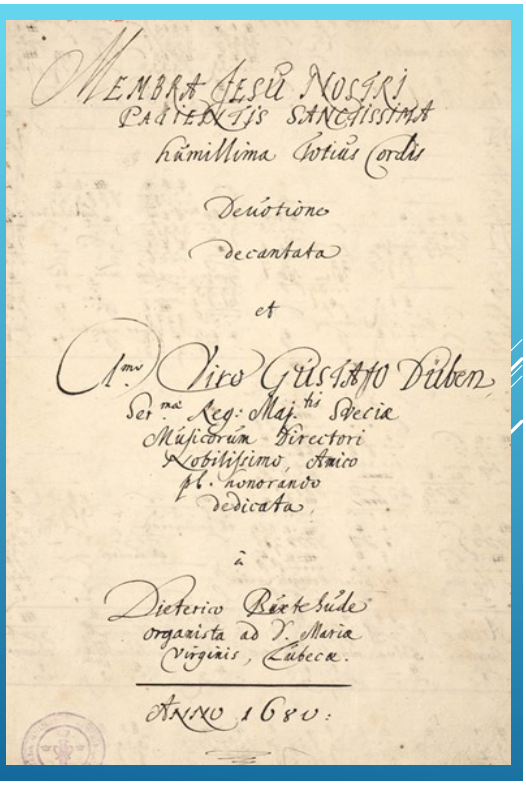
Second thing that's unusual about the chart is that one *[1]* is the first prime number in this chart. Until the early 20th century, many mathematicians and most physicists considered one to be a prime number. It was only in the first part of the 20th century that mathematicians decided that one wasn't a prime number -- because "if we let it be a prime number, it would foul up a lot of other rules." And so *[the number]* one was struck from the list. But for Bach, in a number of moments, one was a prime number, and it comes up in a number of different ways.

Second thing about this chart *[WRB meant the third thing]* is that there are also some letters. And this becomes important -- not for us today, but in much of Bach's work. Various numbers that are prime numbers attach themselves to various letters. And so, when we think of the letters B, A, C, H -- B is 2, that's prime; A is 1, that's prime; C is 3, that's prime; H is 8, and it's not prime. We'll get back to this point, and I'll ask you to remember what I just said. It'll come up.

8'58" Slide #5

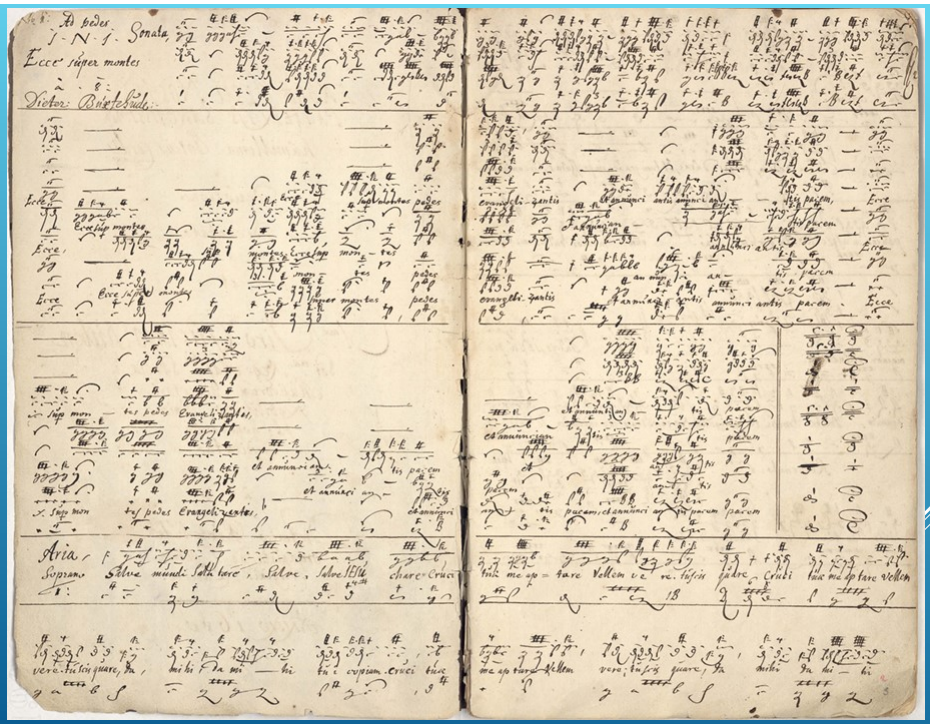
There's another breakdown of the number-letter coding system that has all of the letters -- and most of you are too young to know what that is on the right side, so I'm not going to discuss that any further.

1680
DIETRICH BUXTEHUDE (ca.1637-1707)
MEMBRA JESU NOSTRUM, BUXWV 75
FIRST THREE MOVEMENTS



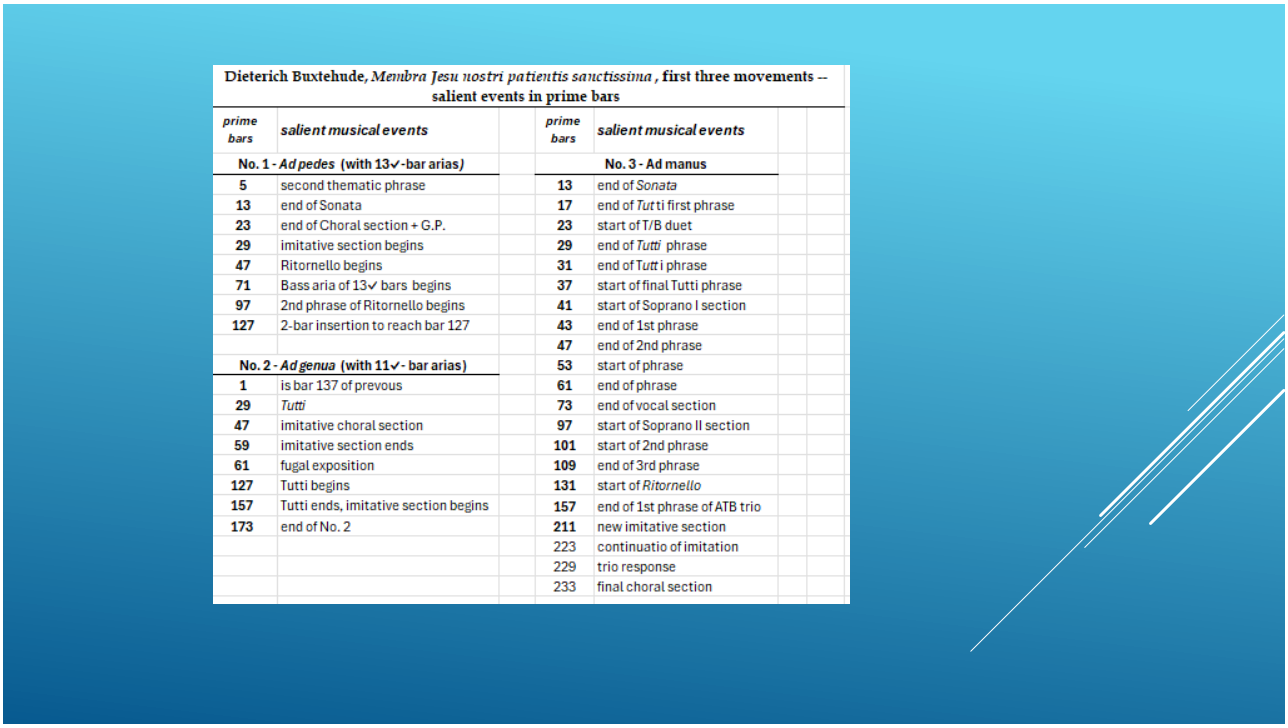
9'20" Slide #6

And there is Buxtehude. This is the title page of a cantata. It has 15 movements, and I'm going to give you a breakdown, a summary of the first three movements according to the location of salient musical events that are in prime-numbered measures.



9'45" Slide #7

I'm glad that you can read that notation. *[audience chuckling]* I won't dwell on the point. Apparently, they could. I certainly cannot. As your eyes dim and refocus, you notice they are in groups. Apparently, those correspond to the measures, and that's about as much as I know about that.



10'12" Slide #8

I wish that I could play this example for you and hold up a card that says "prime." But I am not prepared to do that today. But in the first section of this cantata, all of the short arias by the various soloists in the group are in 13-bar segments. They are 13-bar arias, and 13, of course, is a prime number. If you haven't figured it out by now, a little check mark by a number is my little code that means "this is a prime number" -- notice that. The mathematicians use a check mark for something else, but I ignore that. And the check mark is my mark that means, "hey, this is a prime number."

The second section of the cantata, the arias are in 11-bar segments, interestingly enough. So, what happens in this cantata is that the piece proceeds and each section, virtually every section of the cantata, either starts or ends in a prime-numbered bar. Typically, it starts in a prime-numbered bar. For example, in bar 29 of that first section, the first seriously imitative section begins. In bar 47, a ritornello begins. Bar 71, bass aria having 13 bars begins. And so forth … if I may say that at this moment.

This is likely beyond coincidence, especially when we continue the process and look at the second part of the cantata and notice where the major sections begin. *[points]* A choral section, if you will, exposition. The first *Te Deum* of the second cantata is in a prime-numbered bar. And then the third section of the cantata, which is outlined here . . . the process continues.

Virtually every portion of this work begins in a prime-numbered bar. This would not be apparent if you were listening to it because you don't keep track of prime numbers while you're listening to music. You don't keep track of bar numbers while you're listening to music. You would simply notice what is important, or what grabs you, or what is loud, or what is soft, or what is a change of texture, or a new theme. But all of those events in the mind of many composers of our Western tradition were placed by those composers in prime-numbered bars.

It's perhaps not surprising that no one has noticed this until now, but if you think back to those silos, the prime-number people were buried deep over here, and the music composition people were buried deep over here and -- as I say -- they never got together. No one has ever noticed this, and only one person has ever written anything about it . . . I'm just going to have to go back . . .

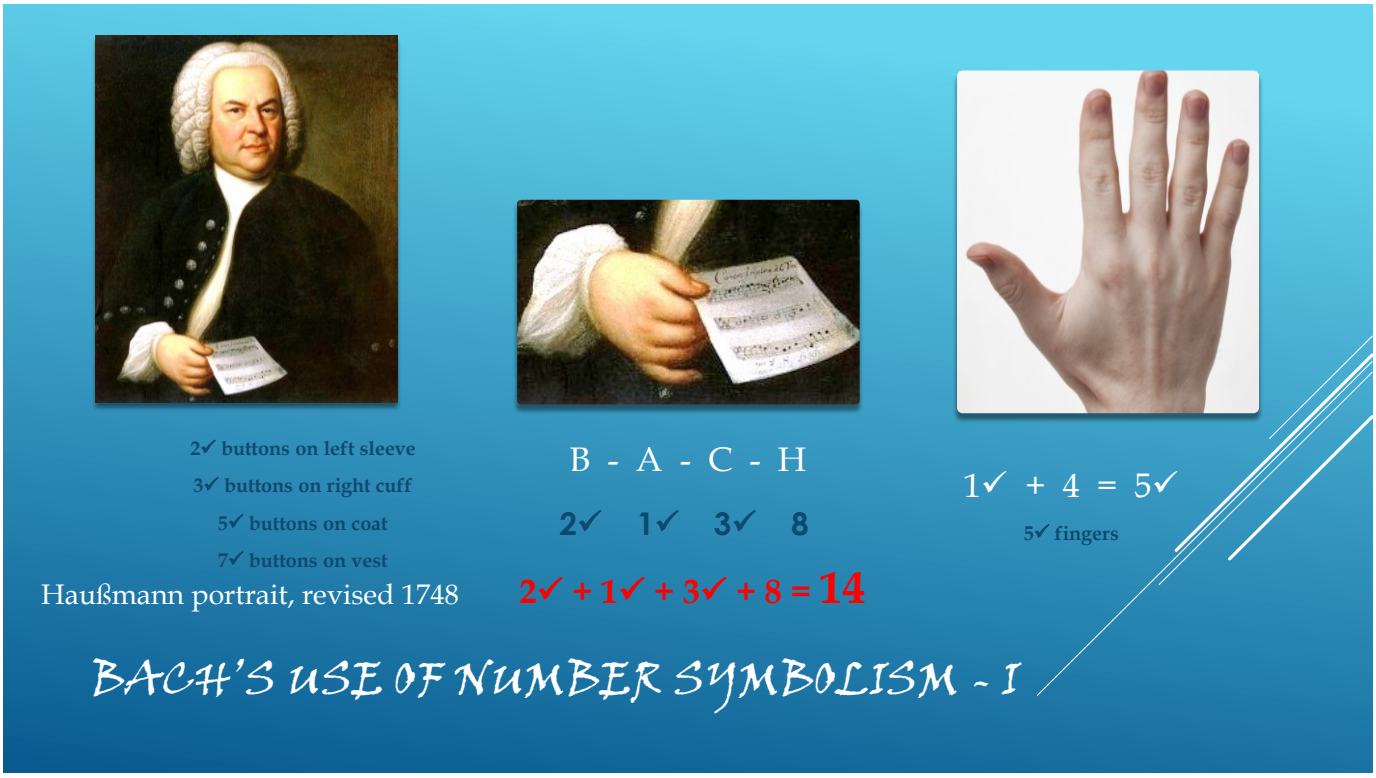
14’10” *[returns to* Slide #3]

. . . and look at this list again. These are musicologists in the audience. *[WRB walks forward to audience]* What do you notice about the people relationships in this group? Who was the teacher of whom? Who had people in common? Who was Beethoven's teenage orchestra partner in the city of Bonn, Germany? *[audience member]* Well, Beethoven couldn't even find his composition teacher? *[WRB continues]* I did not ask. . . I'm asking all sorts of loose rhetorical questions. *[audience member]* So we're not supposed to respond. *[WRB continues]* Who . . . you . . . yeah, these are rhetorical questions. *[audience member]* OK.

With whom did Beethoven share a desk in the court orchestra in Bonn in Germany? Clue: he was a professor of composition at the Paris Conservatory in later life. Are there any woodwind players? *[looks around]* No woodwind players . . . *[WRB finds one]* one woodwind player. Who is the generator of the woodwind quintet literature? Who is the most important one in the 19th century? *[audience member, quietly, “Reicha.]* Antonin Reicha. If you play woodwind quintets, you play Antonin Reicha. Reicha was a Czechoslovakian, sent to Germany because he thought he'd get a better … his parents thought he'd get a better education in Germany. As a teenager, he played in the orchestra with Beethoven. They were exact contemporaries.

As life continued, Beethoven went to Vienna. And Reicha got interested in mathematics, went to Hamburg and studied -- and eventually got to Paris. And took music instead of math as his specialty, became the head of music theory at Paris Conservatory and taught an entire generation of French musicians. Hector Berlioz, and there's the connection we were looking for. Hector Berlioz was one of his students. Like many students, he said, “I didn't learn much from him.” *[laughter]* That’s the connection.

[moving forward through slides] Did you learn anything about notation? Never mind. *[laughter]*

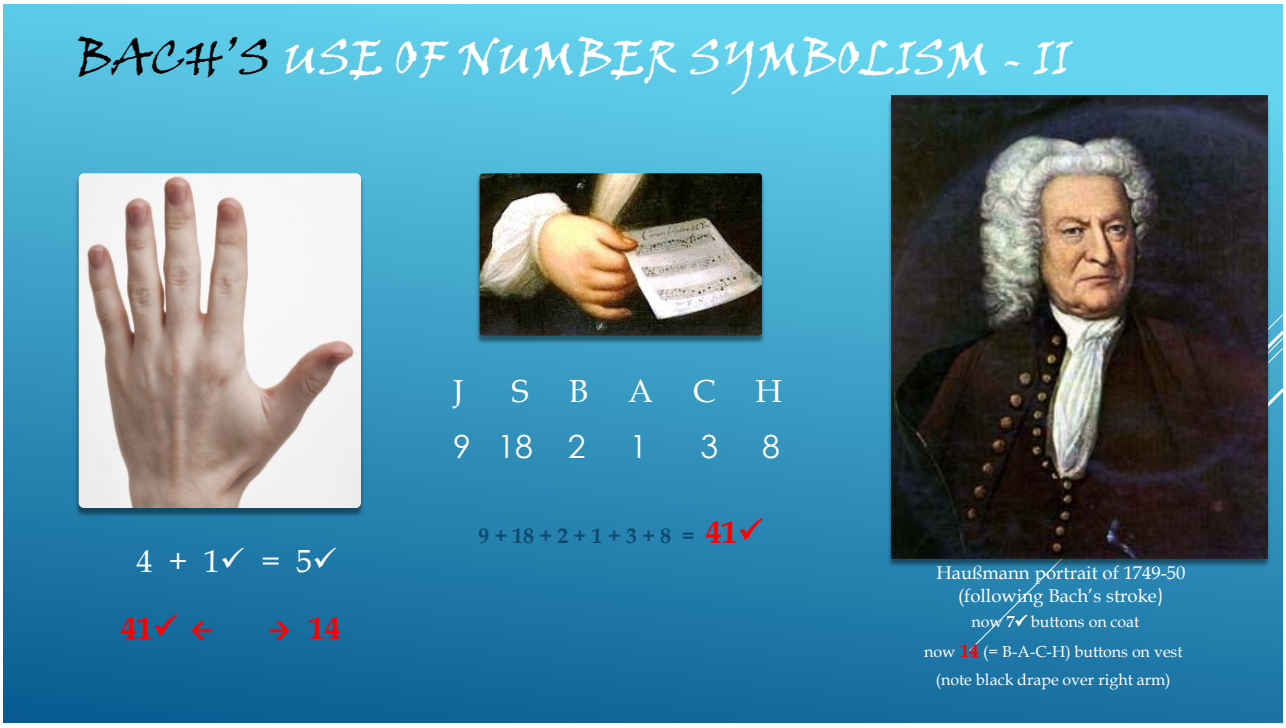


16’50” Slide #9

Let's do a little Bach before we get to Bach. Look at the portrait on the left. There are two buttons on the left sleeve, three buttons on the right cuff, five buttons on the coat, seven buttons on the vest. There's a careful portrait painter!

And there in the middle *[of the slide]* is the B-A-C-H that I was talking about earlier. And we notice that 2 plus 1 plus 3 plus 8 is equal to 14. 14 is an important number in the compositions of Bach. Often, if you look at the 14th measure of a piece, you will find something special happening. Sometimes it's not in the 14th measure, but in the 41st measure, the palindrome of that. Sometimes it is the notes B-flat, A, C, B-natural -- Bach's name in German notation. Or it might happen in bar 144, which is close enough to 14 -- but it seems apparent, at least it seemed apparent to Bach.

On the right *[of the slide]* there is a hand, like your hand. One plus four is equal to five, five fingers. Hold up your right hand. That's the one you're looking at on the screen.



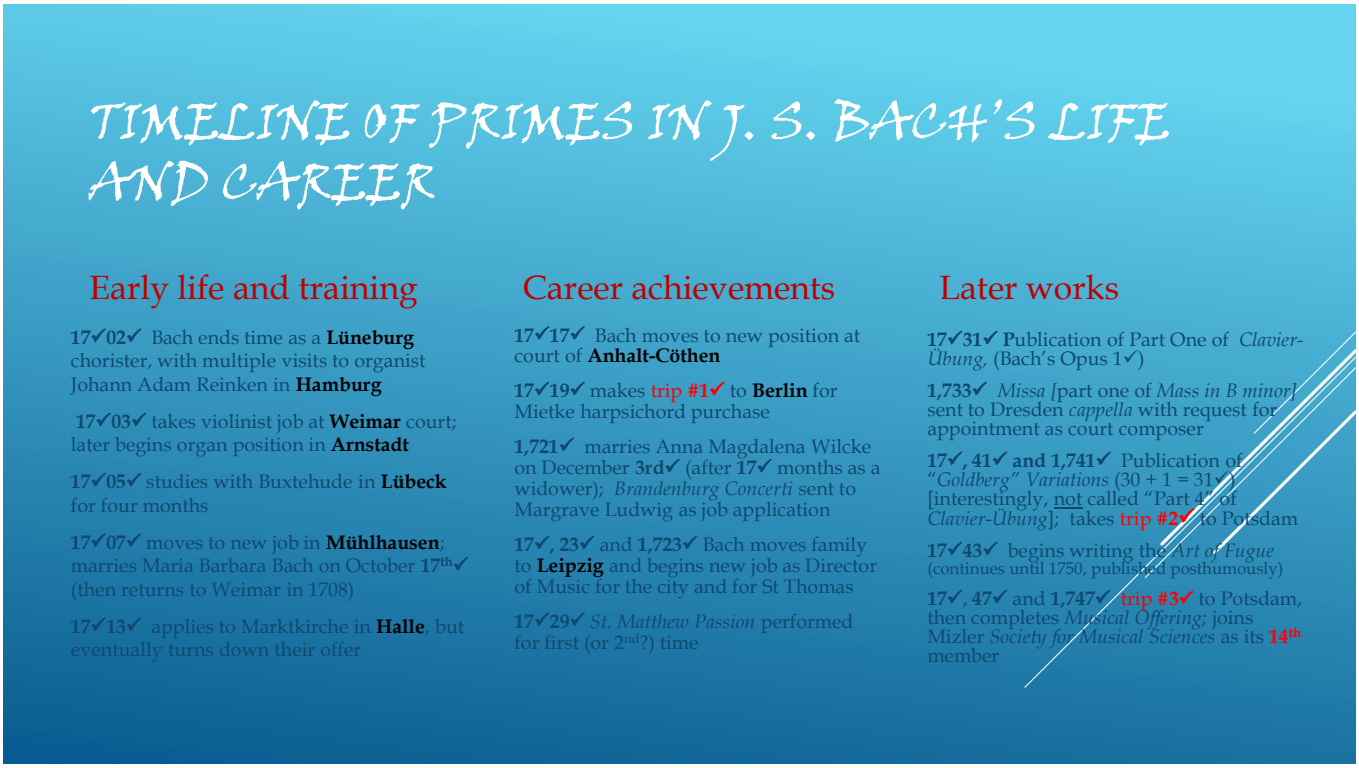
18'23" Slide #10

Now, look at your other hand. Most of us have two hands, and that's the reverse of it. And in red below that, the reverse of 14 is 41.

Now, all of that seems just like playing with numbers. That's third and fourth grade. But this was taken very seriously in 18th-century thought. So, it's not surprising but we pay attention to it. And in the middle graph, we see *[the letters]* J, S. And in that number system, those are 9 and 18, and out of all, they are 41. This seemed very important to Bach.

The right hand *[of the slide]*, he had had a stroke. And you can see, if you look back *[returns briefly to* Slide #9] at his face on the left and now [Slide #10] at his face. What happened?

The careful portrait painter changed the buttons. Now there are seven buttons on the coat. Now there are 14 buttons on the vest. If we look very closely -- I think you cannot see it on the screen -- there is a black cloak over his shoulder *[points carefully]*. And that cloak is symbolic of his medical condition, which was beginning seriously to deteriorate over his last years.



19'53" Slide #11

When we trace out the timeline of Bach's life, we notice -- and I won't read the numbers out for you -- but I will point out that many of the important events in his life happened in numbers which can be called "prime" years. Some of these were under his control, like his wedding dates. Some of these were less under his control, like the dates when he moved from a job to another job. And some of them were completely under his control, as to the year he published the first volume of his keyboard works, the third volume of his keyboard works . . . I've said the wrong number . . . the fourth volume of his keyboard works, and the year that he went to visit Frederick the Great at Potsdam.

20'55" Slide #12

Here's the Musical Offering of Bach *[Ein Musikalisches Opfer*, BWV 1079*]*, one of the group of late works that we talk about when we talk about Bach's late works. In that collection there are two *ricercares*, five canons and five more canons, and then an instrumental sonata (which was one of the chamber works that I played in Germany with that group of students – marvelous, marvelous music experience) and a total of 13 pieces. You can see how many of those are prime numbers, including the five canons and another five canons. When the Musical Offering is spoken of, most people speak of the ten canons in the Musical Offering. However, when you look more closely at the manuscript and the printed version, you find that five of them are numbered and in one group, and the other five are scattered through the entire manuscript in various places. So, Bach was clearly defining a group of five and another group of five. That fits in with my thesis and what I believe was his procedure, of paying attention to prime numbers whenever possible.

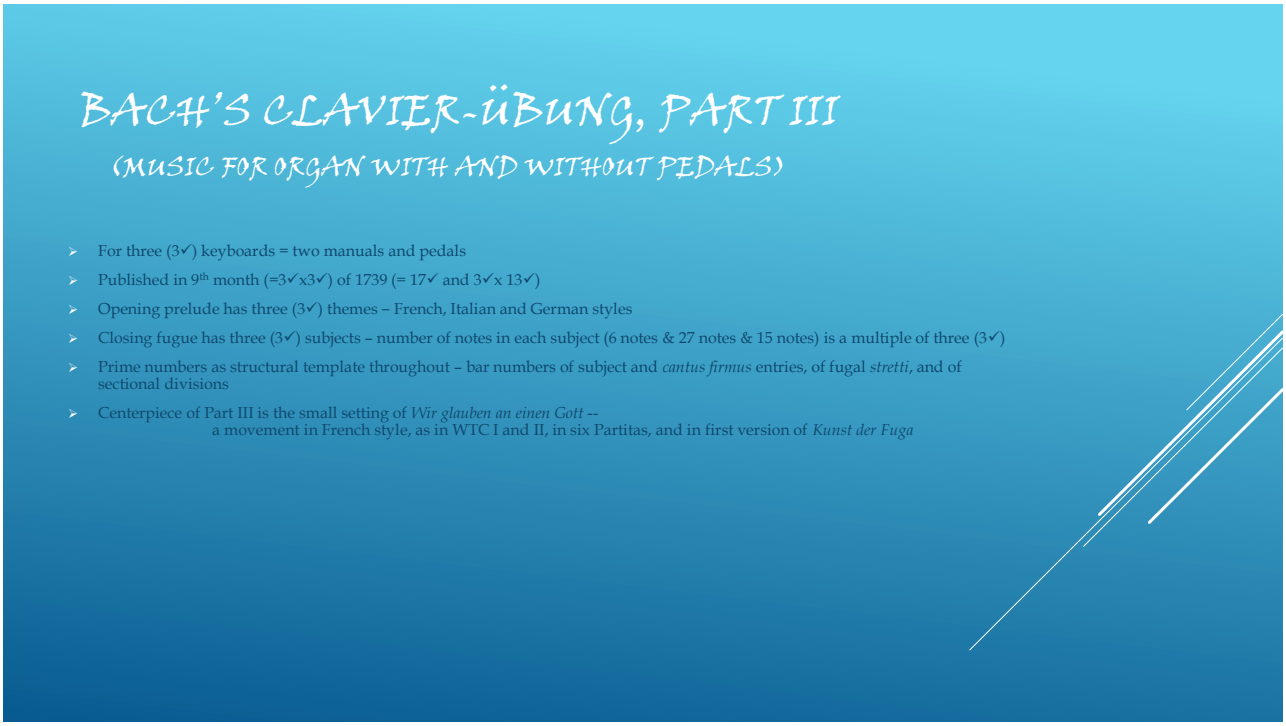
[moves into lines of Slide #12]

He went to visit Frederick II -- Frederick the Great, we usually call him -- on May the 7th, 1747. That was by his design. He had been invited several times and finally accepted the invitation, very carefully *[arriving]* on the seventh day of the fifth month, and -- you probably have not remembered -- but 1,747 is also a prime number. When he published the completed Musical Offering two months later, he published it on the seventh day of the seventh month, still in 1747. And, unusually, in that printing he published the entire date of the composition … of the publication. Usually when you publish something you give the year. Sometimes you might give the month. He gave the entire date, calling attention to that particular date. And, I believe, leaving it to later generations -- such as musicologists gathering in north central Florida -- to discover that these are all prime numbers.

20'35" [continues opening successive portions of Slide #12]

"*Ricercar*" -- old-fashioned name for a fugue. and one that Bach very seldom used otherwise. There must have been some reason for that. He was trying to get Frederick the Great's … get his goat, get his ire up a little bit. He was needling him, in other words, and he set these various little prime-number clues throughout that manuscript. When we remember the number code, we remember that "R" is 17 and "C," you already talked about, "E " is 5, and so the only letter in the word "*ricercar*" that is not a 'prime' letter is the "I." So clearly, it seems to me, Bach was pointing toward the prime numbers as embodied in the number-letter code when he chose the name *ricercar* for two of the keyboard fugues that are in that.

If we could spend an afternoon talking about the Musical Offering, I would have us count the number of notes in each theme of the Musical Offering -- the Musical Offering "Royal Theme," and all the subsidiary themes -- and we would discover that they are in segments of prime numbers. There are 47 notes in one place. There are three notes and five notes and seven and so forth. So, the composition, which is where I began really in 2013 to discover some of this information -- Bach was paying attention to it, and so we do too.



25'25" Slide #13

The third group of keyboard pieces that Bach published is saturated with threes -- not particularly because three is a prime number, although it is. But Bach took hold of the idea that this was Part III of his 'keyboard practice.' And so it was for the organ, an instrument with three keyboards. He published it in the ninth month. The opening prelude, as some of you may know, has French- and Italian- and German-style themes. The last fugue is a triple fugue with three subjects. And if we count the number of notes in each of those subjects, we discovered that, by golly, they are all multiples of three. And so Bach was less concerned with the prime numbers in this composition than he was with the multiples of three, and continued in that same fashion.

26'31" Slide #14

Here was the grand design. And most people don't present this this way, because it does include the conjecture on the far-right side. Bach published his first group of keyboard pieces in 1731 for one keyboard. The second group of pieces for two keyboards. The third group of pieces we just talked about, the *Clavier-Űbung,* Part III, for three keyboards. The Goldberg Variations, for two keyboards.

Interestingly enough, on the title page of that part, that -- oops, I almost said it wrongly -- that fourth piece of keyboard music, Bach does not say 'part four' of the *Clavier-Űbung*, because four is not a prime number. He ran into himself. The first one was a prime number, second one, so he could say 'a second part.' Third one was a prime number, he could say 'third part.' But when he came to the fourth one, he couldn't say 'fourth part' because he avoided the prime number, because four was not a prime number, and he avoided it. And it is my conjecture -- to make out the full grand design of Bach's own publication of his keyboard works -- that the Art of Fugue, which I consider a harpsichord work, was again for one keyboard. And so, this pyramidal structure matches several other places in Bach's output.

28'15" Slide #15

And so here comes the second musical example for today. In the last two years of his life, Bach was summing up his career. He was assembling the score of the B-minor Mass *[Missa H-moll,* BWV 232*]*. He was writing A Musical Offering in 1747, so that's a little more than two years. And he was completing an earlier project that had begun as a counterpoint exercise with his son Wilhelm Friedemann, in 1738-9, right in that era. The two of them were doing counterpoint together, and he had begun a counterpoint exercise with those notes *[goes to piano and plays four notes]*. And that's as far as it went at that point. A couple of years later, three years

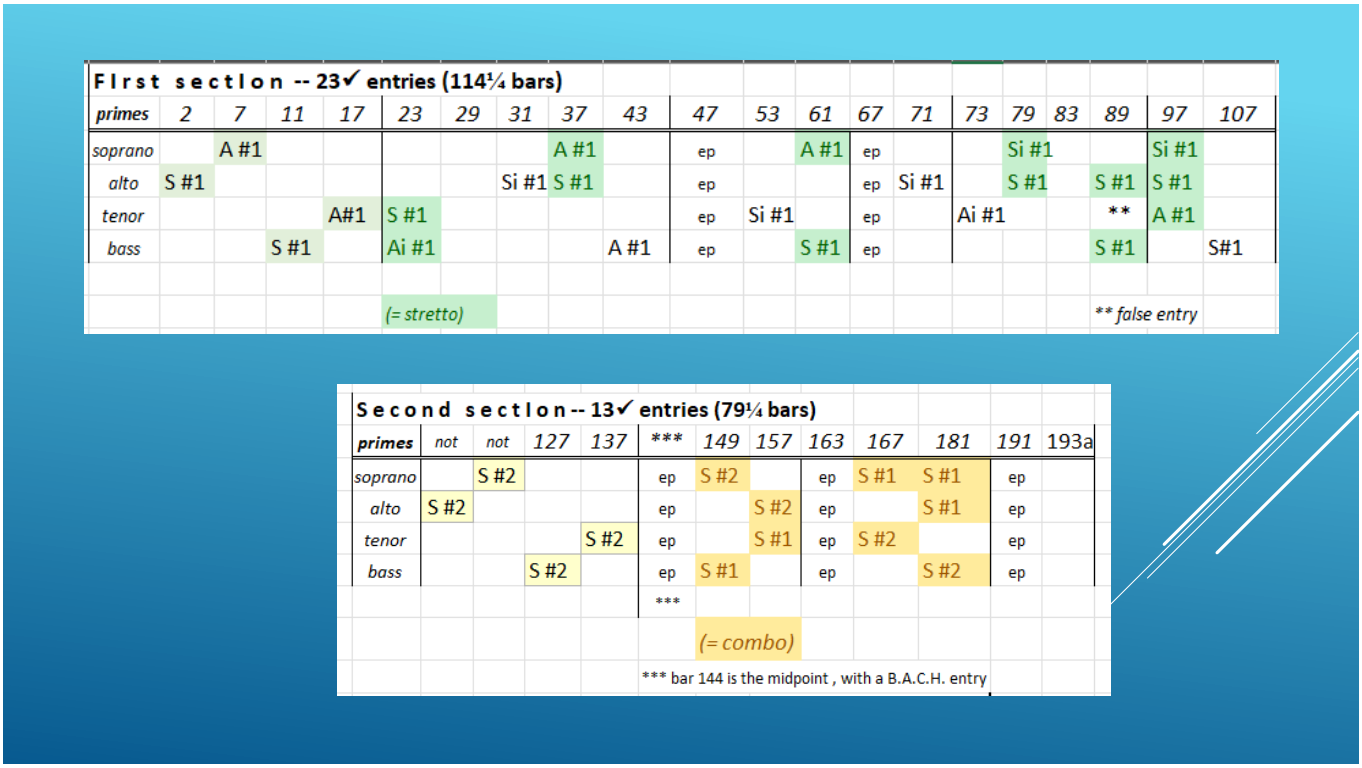
later, he wrote a set of 12 fugues on a subject that began with those notes. And then came back *[later on]* to that group of 12 fugues and revised it thoroughly, and almost finished it in about 1749.

You're looking at the manuscript of the last page of the Counterpoint number 14 of the Art of Fugue, where Bach had combined the first subject of the quadruple fugue *[points]*, the second subject of the quadruple fugue *[points]*, the third subject of the quadruple fugue *[points]*, which was the musical notes B-A-C-H. And the fourth subject, he didn't quite get there. And it took musicologists and music theory people 130 years to figure out that the fourth subject of this fugue was to be the Art of Fugue subject. It's so obvious to us, it is so apparent to us that that's what Bach was doing. But it took musicologists, as I say, 130 years to figure out that that's what was going on.

Here is the manuscript, as I said. You're looking at. . . this *[points]* is the introduction of the beginning of the second subject. Here *[points]* is the Art of Fugue subject *[in augmentation, the first subject of Contrapunctus XIV]* in the bass voice, and here *[points* is the B-flat, A, that's C -- and remember we're looking at keyboard music that has the right hand in soprano clef -- so that is C, then B natural, and that's the beginning of the third subject.

And there's a certain poignance . . . *[31'15" -- projection screen goes dark]* - I don't know what that is, and I hope Kaylee can do something, because I didn't touch anything. *[Kaylee Bagley comes from audience and quickly repairs display]* Brava. *[applause]* Appreciate that.

[WRB continues, with relief] That was the ghost of Bach! Because at this moment he died. *[strong laughter]* And his son *[C. P. E. Bach]* came along and wrote this nice little note at the bottom, which says, as you can see, "at this point where the composer wrote the notes B-A-C-H in the countersubject, the composer is died, is dead" -- that's bad translation -- "he died, he died." So much for the poignance of that.



First section -- 23✓ entries (114¼ bars)

primes	2	7	11	17	23	29	31	37	43	47	53	61	67	71	73	79	83	89	97	107
soprano		A #1						A #1		ep		A #1	ep			Si #1			Si #1	
alto	S #1						Si #1	S #1		ep			ep	Si #1		S #1		S #1	S #1	
tenor				A#1	S #1					ep	Si #1		ep		Ai #1			**	A #1	
bass			S #1		Ai #1				A #1	ep		S #1	ep					S #1		S#1

(= stretto)

** false entry

Second section -- 13✓ entries (79¼ bars)

primes	not	not	127	137	***	149	157	163	167	181	191	193a
soprano		S #2			ep	S #2		ep	S #1	S #1	ep	
alto	S #2				ep		S #2	ep		S #1	ep	
tenor				S #2	ep		S #1	ep	S #2		ep	
bass			S #2		ep	S #1		ep		S #2	ep	
					***							

(= combo)

*** bar 144 is the midpoint , with a B.A.C.H. entry

32'15" Slide #16

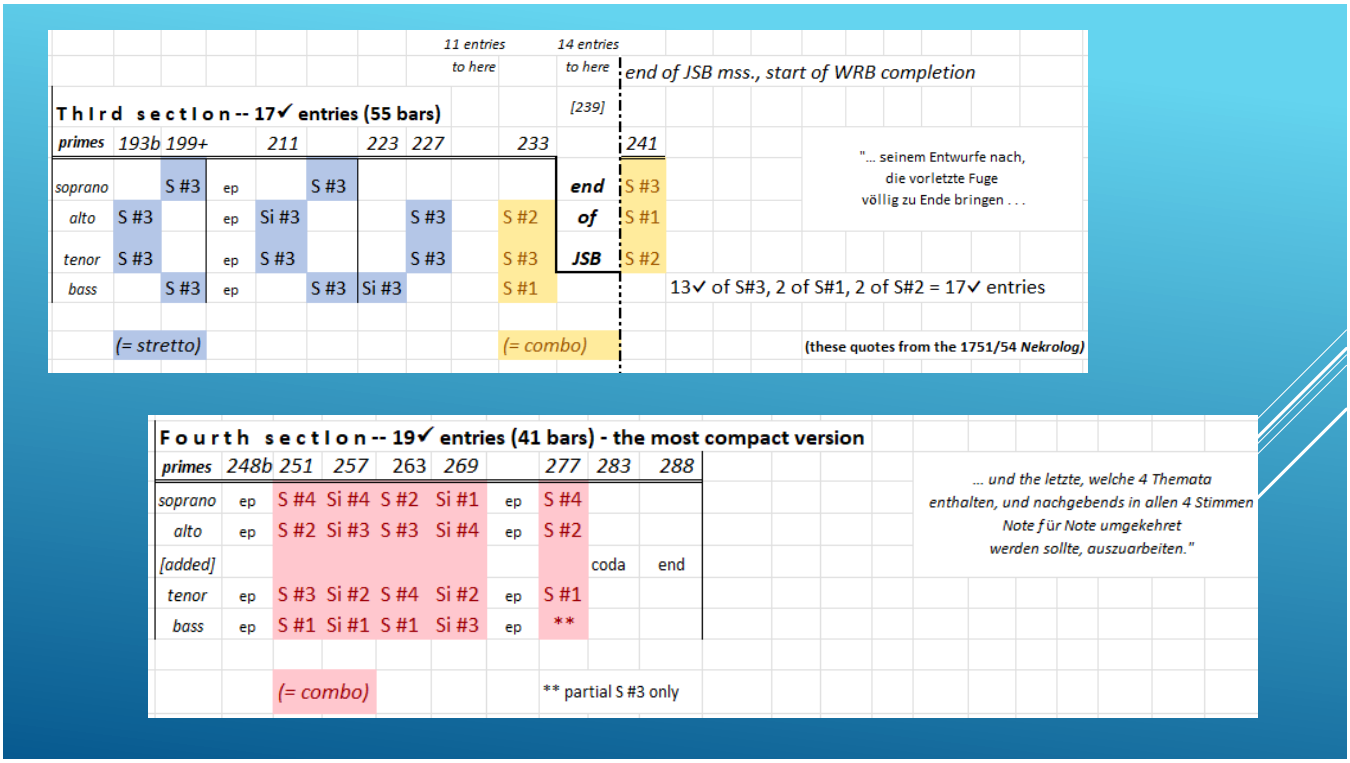
Let's look at the chart that shows the structure of Counterpoint 14 in the Art of Fugue. Look at the way I've got all upon a chart. This is the first section and section -- second section -- the next slide will be section three and section four.

Here *[points]* are the prime number of measures in the second section. So, you see where the chart is. And here *[points]* is the fugal structure. Subject One comes in the alto voice, comes in the soprano voice, the bass voice, the tenor voice -- in bars 2, 7, 11, 17, and those are prime-numbered measures. I do not need to say to this audience that this pattern continues.

In the next section, in bar 23, Bach begins to write strettos on that first subject. A little while later, he does a triple stretto. It begins around bars 31 and 37, which are prime-numbered measures. And that's a rather large stretto there. He tucks in one more entry in that section. It has an episode in bar 47, goes on and eventually gets to a third exposition in bar 53, a fourth exposition beginning in bar 71, and so forth. A total in that first section of 23 entries of the subject. That goes to bar 114 ¼ bars, because the music continues until exactly that point until the second subject comes.

In the second section of this fugue, the first two entries are not in prime bars. There's a huge gap between 113, which is a prime number, and 127, which is a prime number. So, it was not possible for Bach to put a subject entry in those prime-numbered bars. But by the time of bar 127 and bar 137, he was back in groove, and that was able to be continued.

In this second section of this counterpoint of the Art of Fugue, there are 13 entries. 13 is a prime number. And if you do a quick math, you look at the length of the first section, 114 ¼ bars. And the length of the second section, 79 ¼ bars. And you establish the proportion between those two. And the proportion between those two numbers is 1.441 -- that you can do on the yellow pad. I doubt if most of us would bother; we would reach for our calculators. But Bach could apparently do it, and that is far beyond the level of coincidence.



35'48" Slide #17

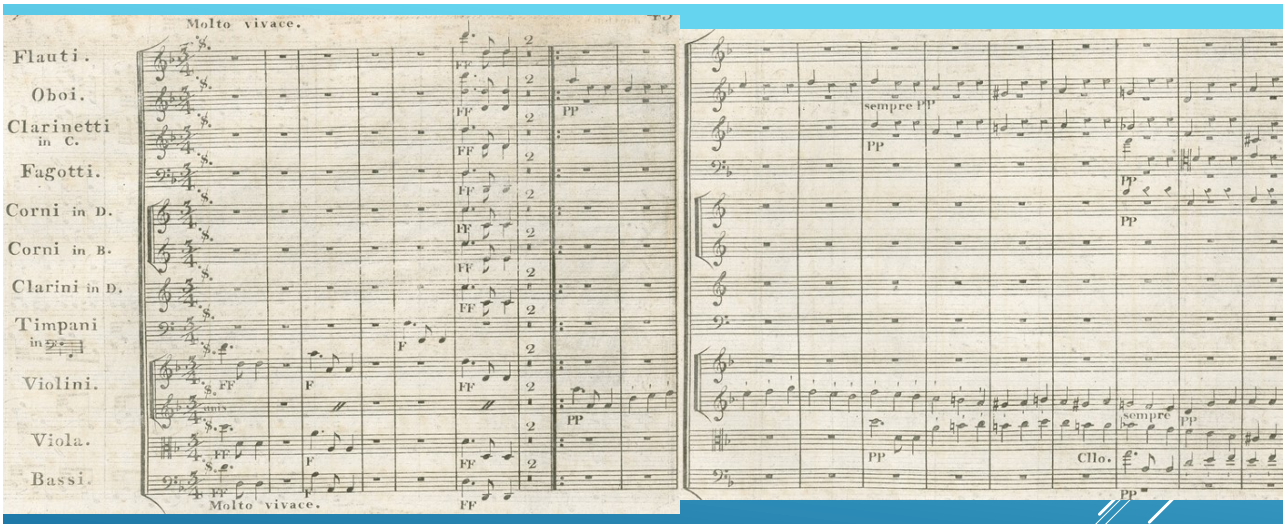
We're continuing now into the third section of Counterpoint 14 of the Art of Fugue: third section, and bar 193, the second half of it, and there are the entries in stretto. It is a long subject, and Bach was getting a little antsy, and so he put them *[the subjects]* in stretto from the very beginning of this section. The chart

continues, and bar 233 is the point where that manuscript began that I showed you. And that's the place where Bach ran out. And that's the place where I have started a completion of this.

And on the basis of the research that I have done, which is based on the research of many people before me, I've added the prime-number speculation thesis to Bach's plan. I believe in some ways (if it's not presumptuous) I have entered into the mind of Bach at this point, and I understand what Bach was trying to do in terms of placing subject entries for the structure of the quadruple fugue in prime-numbered bars. And so, in my completion -- which is for harpsichord, but can be played by anybody -- all of the entries of the subjects, various subjects, happen in prime-numbered bars. I complete in my version *[points]* I add another entry of subjects one, two, and three, as you see in bar 241, for a total of 17 entries in that section -- that's a prime number, no surprise. The fourth section, which is entirely my speculation, is involving combinations of all four subjects. *[WRB walks toward audience]*

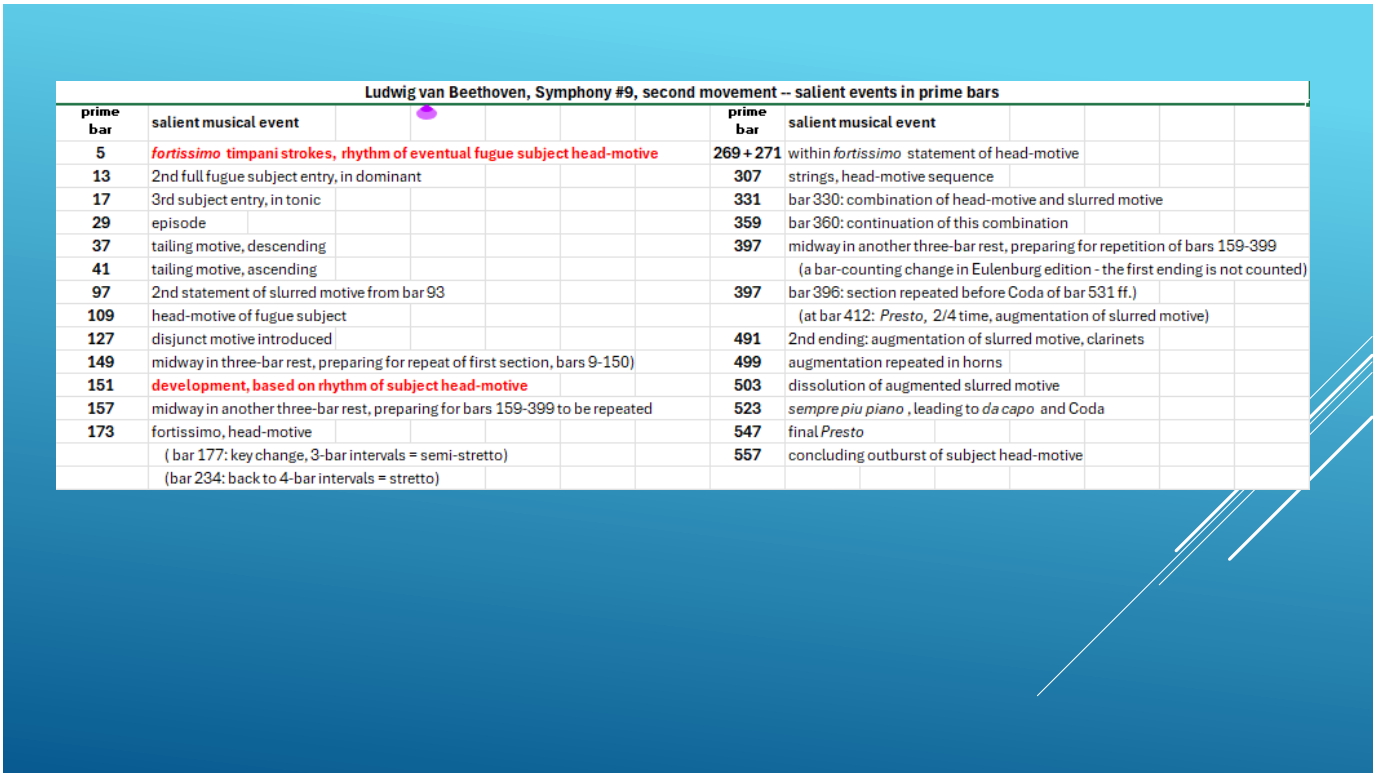
Now, there is -- truth to tell, and I do tell the truth -- there is a cottage industry internationally in writing completions for this counterpoint. You can go online and find a list of 70 or 80 different people who have made completions. Several of them have been published, a lot of them have been recorded. There are several major organists who have not only written but recorded their own completions of the counterpoint. And I don't cast any aspersions on theirs, but my completion is the one that has absorbed and carried out the prime-number organization. It is on the order . . . as compared with the other completions, it's one of the relatively short ones. There's some people that have gone on and on and on. There's some people who have done a little bit shorter than mine, but mine's about there.

I wish I could do a whole afternoon on just that, and talk about myself, but I won't. I'll tell you that in March or possibly April, there will be a radio broadcast from our local classic station, and he *[Dana Hill, host of Magnum Opus]* is going to play a recording of the Art of Fugue, and it'll be one of those that dribbles off at the end, as Bach's manuscript is. And then he will play my completion, and so there'll be the opportunity for you to hear it at some point.



39'50" Slide #18

So much for Bach. The next "B." This is the teenage classmate of Antonin Reicha. This is his Ninth Symphony. By the time he was the leading composer in Vienna and had added the chorus to his ninth symphony. This is the second movement *[sings]* Bump -p -p p -p -p - oh -ppoh -poh. It's that now. And it's a scherzo, unusual for a symphony. Beethoven was shaking the foundations as he moved things around. And so, he put the scherzo second. But interestingly enough, that timpani stroke right here *[points in score]* comes in bar five. And then when the fugue starts, it starts in bar nine. But the subject, the series of subject entries then follow very much a prime-numbered pattern.



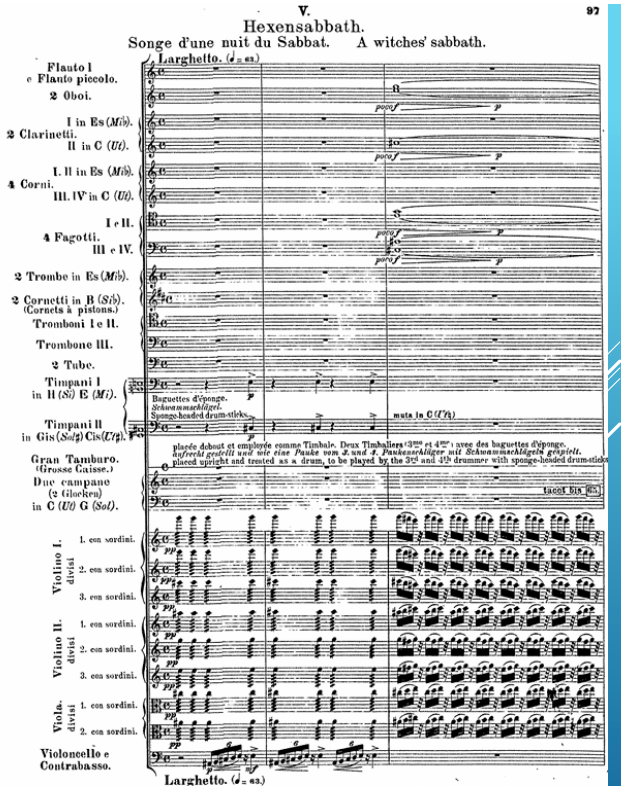
Ludwig van Beethoven, Symphony #9, second movement -- salient events in prime bars

prime bar	salient musical event	prime bar	salient musical event
5	*fortissimo* timpani strokes, rhythm of eventual fugue subject head-motive	269 + 271	within *fortissimo* statement of head-motive
13	2nd full fugue subject entry, in dominant	307	strings, head-motive sequence
17	3rd subject entry, in tonic	331	bar 330: combination of head-motive and slurred motive
29	episode	359	bar 360: continuation of this combination
37	tailing motive, descending	397	midway in another three-bar rest, preparing for repetition of bars 159-399
41	tailing motive, ascending		(a bar-counting change in Eulenburg edition - the first ending is not counted)
97	2nd statement of slurred motive from bar 93	397	bar 396: section repeated before Coda of bar 531 ff.)
109	head-motive of fugue subject		(at bar 412: *Presto*, 2/4 time, augmentation of slurred motive)
127	disjunct motive introduced	491	2nd ending: augmentation of slurred motive, clarinets
149	midway in three-bar rest, preparing for repeat of first section, bars 9-150)	499	augmentation repeated in horns
151	development, based on rhythm of subject head-motive	503	dissolution of augmented slurred motive
157	midway in another three-bar rest, preparing for bars 159-399 to be repeated	523	*sempre piu piano* , leading to *da capo* and Coda
173	fortissimo, head-motive	547	final *Presto*
	(bar 177: key change, 3-bar intervals = semi-stretto)	557	concluding outburst of subject head-motive
	(bar 234: back to 4-bar intervals = stretto)		

40'55" Slide #19

Again, as with the others, I don't intend to talk through the entire chart. You can read the numbers as well as I. But the major events in this -- Beethoven, second movement of the 9th symphony -- do occur, continuing up into the prime numbers. into the 200s, 300s, and 400s, and 500s -- do occur in prime-numbered bars. Again, as I've said two or three times, I think this is beyond coincidence. I think Beethoven was paying attention to the prime-numbered bars as they were going by, when he was working. I think that Beethoven had absorbed something from Reicha when they were teenagers. And Reicha had taught the various French composers about prime numbers in the privacy of his studio. And Beethoven, off in Vienna, had remembered it. And sure enough: when we look at this late work, we find the sequence of prime numbers -- a very clear template. I could make that same statement in entirety about each movement of the ninth symphony and I can make it about the third and the fifth symphony as well Again, we need to take an afternoon or a week or a month to talk about that.

1830
HECTOR BERLIOZ (1803-1869)
SYMPHONIE FANTASTIQUE, *OP. 14*
5TH MOVEMENT, "WITCHES DANCE"



42'40" Slide #20

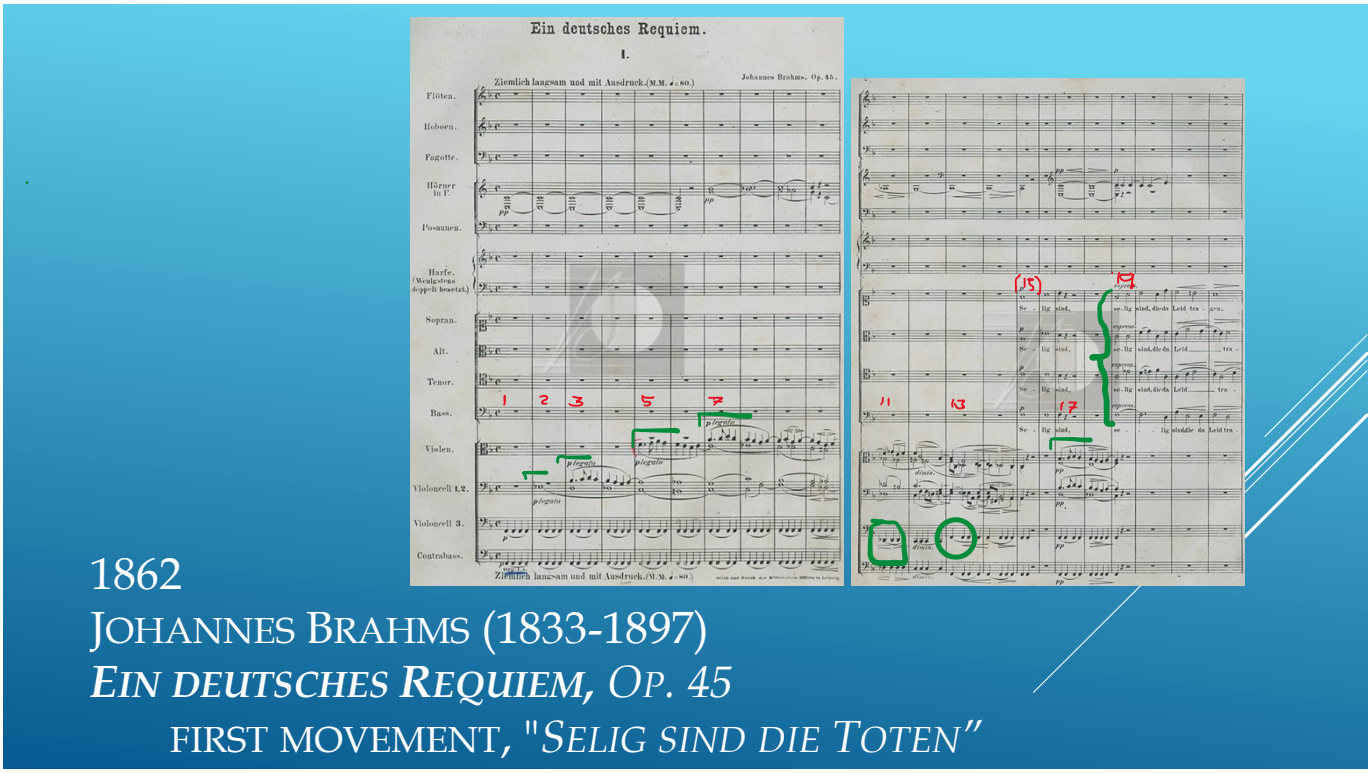
Next "B", Hector Berlioz, the one who thought he didn't learn much from Antonin Reicha *[quiet chuckle]*. Here is the Witches Dance, the fifth movement of the *Symphonie Fantastique*. Again, this composer was playing with the movements -- he had five movements in the symphony. I will not tell you that five has a special significance, but it does. And the fifth movement, among other movements, follows the prime template. You can see just the very beginnings of that in the score. This is bar three, where this new figure happens. And if we were to go on in that symphony,

Hector Berlioz, *Symphonie fantastique* - fifth movement -- salient events in prime bars

prime bars	*salient musical events*	*prime bars*	*salient musical events*
3	winds + strings *agitato* figure	241	***Ronde du Sabbat***
11	*arpeggio* figure	269	end of full orchestra ***ff***
17	winds + strings *sf>p*	293	rising figure
29	Allegro assai	307	sighing figure in woodwinds
41	#63 - Allegro	311	sighing figure repeated
47	piccolo entrance	313	sighing figure repeated
61	#64 - theme	317	sighing figure redoubled
71/73	*ff + agitato* figure	331	strings tremolando over theme in bass
79	rapid descent	347/49	bar 348, **Dies irae** phrases in celli and horns
83	#65 - rapid ascent	359	bar 358, **Dies irae**theme in contrabassi and horns
101/03	bar 102, bell motive		#78, imitative passage, first entry
107	viola solo	367	bar 368, imitative passage, second entry
109	brass + string *ff*	373	imitative passage, third entry
127	***Dies irae*** first phrase	379	#79, imitative passage, fourth entry
157	woodwinds + strings *pizzicato*	409	unison strings
163	***Dies irae*** second phrase	409/19	bar 414, **Dies irae** in unison brasses
239	full orchestra ***ff***	467	orchestral ***fortissimo***
		479	C major coda, fragmented antiphonal winds and strings
		487	#85, bar 486, **Dies irae fragment**
		509	bar 508, orchestra

43'30" Slide #21

fifth movement, we would find that the major events, the colossal events, the *Dies irae*, for goodness -- the beginning of the Witches Dance, the *Dies irae* several more times --- all these happen in prime-numbered bars.



43'55" Slide #22

Next "B." *[Johannes Brahms]* Well, this score has a little analysis on it. So, you can see that the red numbers are the prime-numbered bars. See that the green marks are the entries of musical ideas. And the big green bracket on the right-hand side is the beginning of the theme of the first movement.

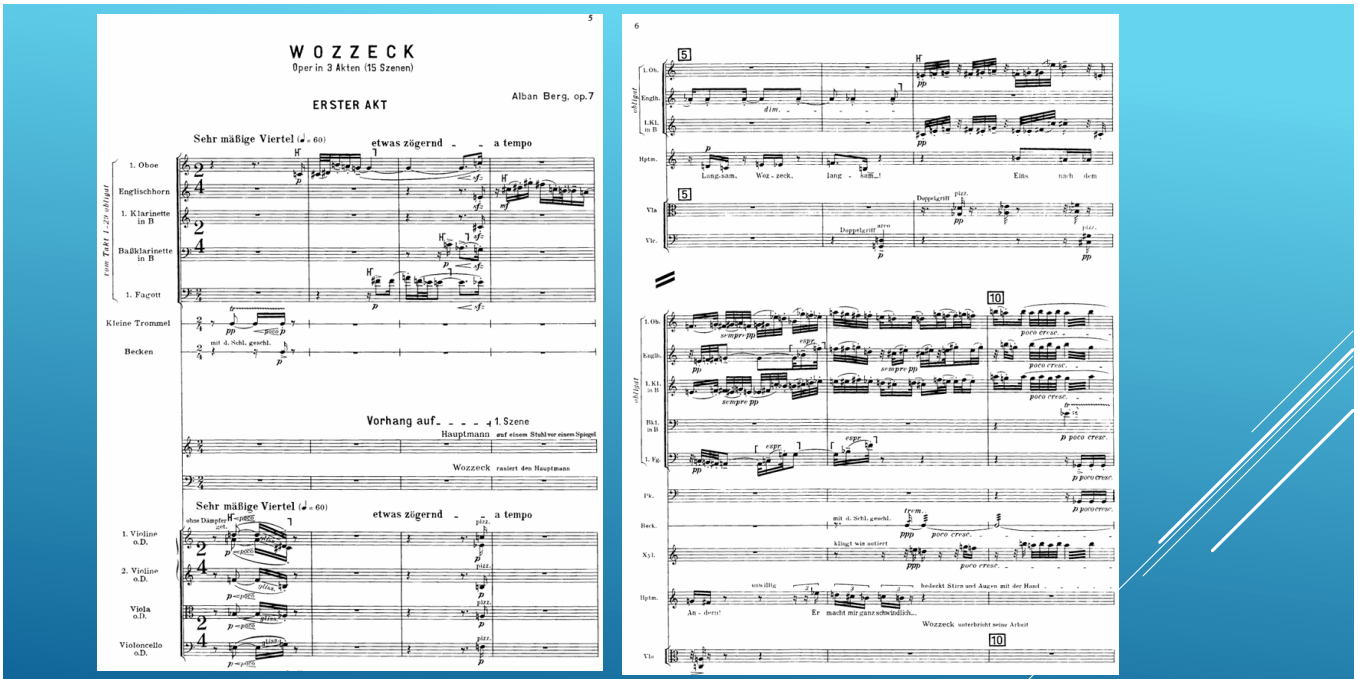
There's a little fake at bar 15. 15 is not a prime number. Brahms fakes. He lets the chorus enter, but it's not the theme. It's just the first three notes of the theme, in augmentation. So, I submit to you that in this musical example, Brahms is giving us a textbook example of how the template of prime numbers may influence and, in fact, direct composition. *[moves to grand piano]* If any of you are string players, you're welcome to hum along. *[WRB plays opening of* Ein deutsches Requiem*, calling out]* Bar two, bar 3, bar 4 – bar 4 was not a prime number, so there was no entry at that point. Do it again *[plays again from beginning, calling out]*. Bar 5. Bar 7, *[Bar 11]* here it is. Bar 15, which is not prime, chorus. Bar 17, orchestra. Bar 19 now-- this is the theme. And so forth *[plays up to bar 29]*. So, textbook example. Again, I'd love to spend more time with you.

Johannes Brahms, *Ein deutsches Requiem* -- salient events in prime bars

prime bars	salient musical events	prime bars	salient musical events
1	Cello III and Contrabass, opening pulsation motive	67	tenor entry of opening theme, cf. bar 3
2	Cello II entry	73	soprano entry of opening theme, cf. bar 7
3	Cello I entry	79	extra inserted bar in paraphrase of opening
5	Viola II entry	89	bar 88: paraphrase of bar 55
7	Viola I entry	97	bar 96: recapitulation of opening (in lowered submediant)
11	D-flat in bass, lowered submediant leading to dominant	101	bar 100: recap of bar 15, still in lowered submediant
13	return to tonic	103	bar 104: development of bar 15
(bar 15)	(chorus sings first three notes of main theme in augmentation)	107	bar 106: new theme in alto voice
17	instrumental response	109	contrasting theme in tenor voice
19	chorus begins first theme	109-113	midway between bars 109-113, restatement of first theme from bar 19
23	sopranos sing 2nd half of main theme	127	as at bar 37 - recap of new theme
29	chorus, continuation of main theme	131	as at bar 57 - melodic climax
31	pitch climax in soprano	137-139	sequential modulation, leading to bar 140 restatement
37	new theme in oboe	149-151	[twin primes] introduction of harps, melodic climax (this passage eventually to be repeated with changed text at end of seventh movement, as ending of entire work)
47	second theme in lowered submediant		
53	pitch climax in soprano	157	coda: tonic harmony to the end
59	end of imitative passage		
61	closing cadence in lowered submediant		**After partial performances of smaller portions during 1867-68, the completed *Ein deutsches Requiem* was premiered in 1869 in Leipzig in its final form of seven (7✓) movements**

48′02″ Slide #23

Here's the chart. There is the list of prime numbers. *[John-Peter Ford signals from audience]* And I'm getting a signal. Am I getting a signal? I'm getting a signal. *[WRB begins to move more quickly]*



48′25″ Slide #24

Next "B" – Berg, *Wozzeck*. I guess I can trace *Wozzeck*. Arnold Schoenberg was the … one of my … my . . . let me start the paragraph again. I studied musicology with Paul Pisk at UT-Austin. Paul Pisk was a student of Arnold Schönberg, *[but was a]* primary student of Guido Adler. And Schoenberg's compatriot -- successor in many respects -- was Alban Berg. And so, there's the connection with Berg.

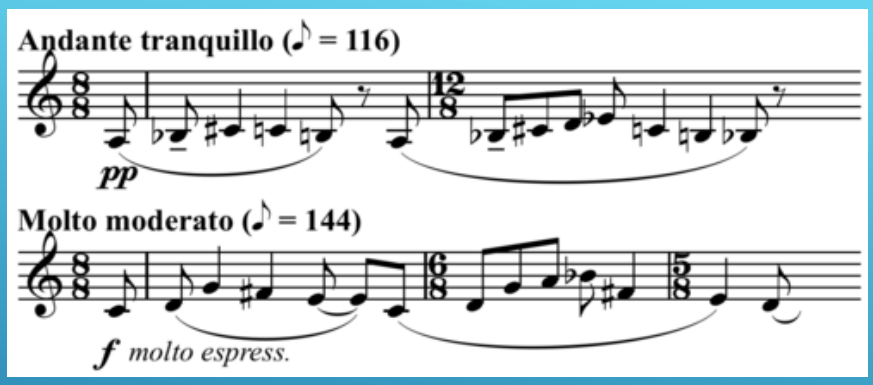
Alban Berg, *Wozzeck*, opera in three acts - salient events in prime bars

prime bars	*salient musical events in prime bars*
1	opening section - Wozzeck shaving Hauptmann
5	voice entry of Hauptmann
11	new tempo
29	new section
41	Wozzeck sings
47	climax
53	change of style
67	Wozzeck sings about weather
73	Hauptmann again
79	climax
83	Hauptmann vocal entrance
97	sings again, after laughter
109	new section
127	Wozzeck entrance
149	new section
151	strong declamation
157	new tempo
173	curtain down - interlude - piano 4 hands
223	new section/tempo - in *Sprechstimme*
227	Wozzeck begins section
233	Wozzeck begins section
239	end of *Sprechstimme*
257	Wozzeck entrance (spoken)
263	Andres entrance (spoken)
269	climax
271	new section
[275]	"highest fear"
281	new section
283	final bar before big G.P.
293	"brightest sunlight" - "heaven"
307	curtain closes after end of singing

prime bars	*salient musical events in prime bars*
317	solo clarinet
331	military music approaches
337	Drum Major in front of window
359	"I am the respectable person"
379	end of section
419	horn/trumpet calls, out of meter
443	new tempo/section
339	climax
467	Broadly
487	end - bar 488, curtain up
503	2nd variation
509	bar 510 - 3rd variation
523	bar 524 - 5th variation
541	"Dunkel"
547	"Doctor"
557	"Ringe" (mushroom rings apparition)
569	14th variation
577	Doktor begins phrases "Wozzeck"
587	"Hauptmann"
593	Doktor entrance
619	bar 20, 20th variation
641	section
643	section
647	curtain falls (bass clarinet)
653	final woodwind notes
659	main and secondary themes
673	highest note - "keiner"
677	cantabile (Drum Major) entrance
683	end of phrase
691	word "Tambormajor"
709	Marie entrance
End of Act One	

49'10" Slide #25

If we go through *Wozzeck*, which is quite a task, *[laughter]* we discover that the events, the salient events in *Wozzeck*, happen in prime-numbered bars.



1936
BÉLA BARTÓK (1881-1945)
MUSIC FOR STRINGS, PERCUSSION AND CELESTA, SZ. 106
FIRST MOVEMENT (FUGUE)

49'25" Slide #26

Last "B" – Bartok, *Music for Strings, Percussion, and Celesta*. The first movement of the piece, of the work, is a fugue. The second movement is a sonata, sonata allegro form, and the third movement is essentially a rondo. So, Bartok was playing with musical forms.

Béla Bartók, *Music for Strings, Percussion and Celesta*, first movement, fugal structure -- salient events in prime bars

prime bars	*salient musical events*
1	first subject entry
5	second subject entry
13	fourth subject entry
17	fifth subject entry
(27)	stretto of outer voices
31	episode
37	end of episode
47	bass entry of head motive sequence
53	timpani entrance
59	bass/cello entry of head motive sequence
61	bass/cello entry of inverted subject
67	end of contrabass section before muted strings
73	1st violin entry of inverted subject
79	2nd bar of first celesta passage
83	bars 82/87 coda = direct/inverted head-motives
89	bar 89 of first movement is 1st bar of second movement
521	bar 521 of second movement is 1st bar of third movement

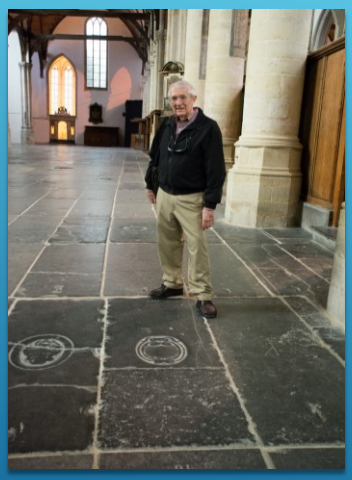
(1) Within movements, primary sections begin in prime measures.
(2) Each movement begins in what would have been a prime bar in the preceding movement.

49'50" Slide #27

And when we look at the first movement, which is the fugal one, we discover that the primary events, the entries of the subject, the important moments in work happen in prime bars.

JAN PIETERZOON SWEELINCK IN AMSTERDAM

the Oude Kerk



Sweelinck's tomb

Sweelinck's bust

50'10" Slide #28

Well, we don't get the travelogue today -- because you only scheduled me for an hour. But I am wondering if I have raised any points that you can answer, that you could ask that I could answer, or if you are saturated and don't want to hear another thing about prime numbers. *[looks around for a question]*

Thoughts? Yes. Ryan. *[audience question]* So my mind goes to how composition is taught. It's phrases of fours, eights, and sixteens. [*WRB* Right.] And that was described as early as Hanson Wright as "a fetus on melody." And I'm wondering if, and also, you know, describing it in modern texts . . . *[WRB* Yeah, yeah.] I'm wondering if you view the idea of symmetrical phrasing in groups of 4, 8, and 16 as being contrary or complementary to what you're describing as primary events, or influence.

[WRB answers] What Reicha said about that to his students in the mid-19th century in Paris was "four bars and plus four bars is all well and good, but wouldn't it be more interesting if we use prime-numbered lengths?' unquote, paraphrased. There's a fake that can happen if you're looking at prime numbers and you have four bars and the next one *[bar]* is five. That's prime. You look at 12 *[bars]* and the next one is 13. That's prime. You look at 16 bars, that's *[the next one is]* 17, and you could probably do this prime analysis for any rock tune on the planet, because they would be 4 – 4 – 4 – 4 - 4. That's, as I say, it's a fake. So, I usually disregard any correspondence in the first 20 *[bars]* or so *[when searching for]* prime numbers. I notice them, but I disregard them. It's only after we get to bar 29 and bar 47 and bar 61, then I start to say, 'aha, this is likely following the prime sequence.' That's a longish answer, but that's my answer.

Other thoughts? You want the travelogue? Yes. *[audience question]* I want to go back to your completion of the Bach, because sometimes the completions, sometimes, and it reminds me a lot of, maybe (help me with the name) Harvard professor Dr. Robert Levin . . . *[WRB]* Yeah, Robert Levin. *[audience member]* . . . who has done a lot of Mozart completions. And when you're looking at completions, and specifically in that Bach, and looking at the prime number structures, how else are you looking at the completion besides just the prime number structure and completing the work itself?

[WRB answers] It's interesting, I'm not sure how quickly I can find the . . . it may take too long. *[goes backward to Slides #16-17]* There is a proportionality—oops, there it was -- proportionality in the structure of the entries. If we look at the first section, first exposition, see *[points]* the pattern of entries, second section, let's see the gaps, third section -- look at that -- and then when you go down to the fourth, to the fourth section, see *[points]* where all the fourth subjects come. Bach was -- and I believe I am -- completing what is called a "matrix fugue." There is a matrix in which everything works together, and that continues. Again, that's the semester's work. Yeah. So, short answer, is that enough? Yes. Short answer.

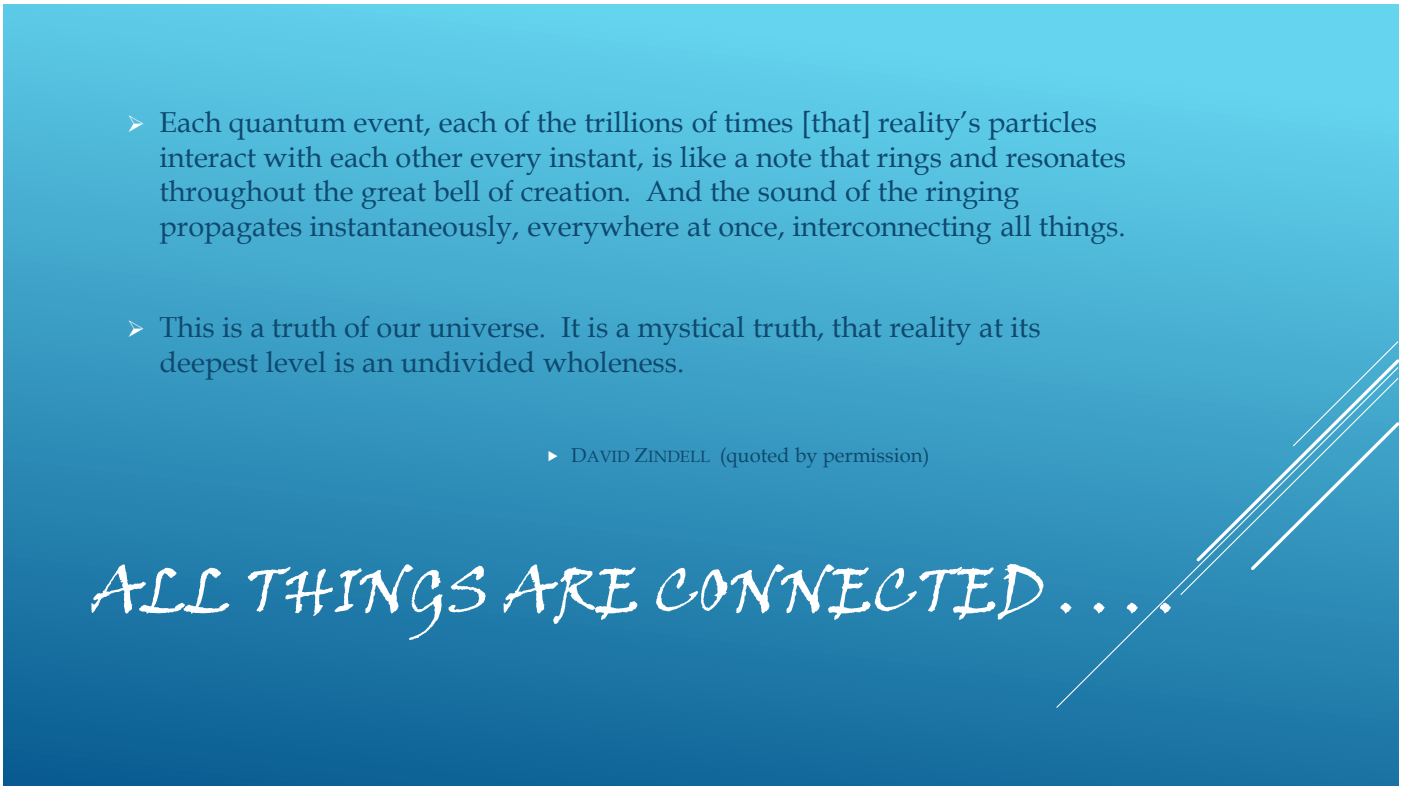
[*Audience member]* Well, and you can also, with like this completion, or like with possibly like another completion, are you looking at other works of Bach to see how he was doing earlier fugues and such as that? [*WRB* Yes. Okay. *[refers to Slide #17]* However, in this, in my completion, so much of that is actually Bach's music, especially this group right here *[points].* It is so tightly constructed that if you put it together right, there's not much room for me. This *[points]* is about two measures here that I wrote, about two measures here, about three measures here, and I think there are two measures here. So, it's very, extremely tightly woven, according to the pattern that I believe Bach was following. So, I can't pretend that I was creating in the mold of Bach. What I believe I've done is diagnose and analyze, de-structure Bach's entire plan -- and then put it together in notes. That's what I believe.

Yes. *[question from audience]* So just say how you'd respond to if somebody were to critique that. For someone looking at the world as a hammer, everything is a nail. *[WRB]* Yeah, that's fair. *[audience member]* What would be your response to that? Because there is a whole bunch of other music, and this repertoire is, however extensive it is, it's really miniscule in relation to the number of pieces that are there. *[WRB]* That's true. *[audience member]* And so if we are picking and choosing, we can find the pieces, You know, Bach was mathematically approached. So was very, very good, dealt with numbers. There was a whole bunch of other composers who would not fall in their music.

[WRB answers] What I have said at the beginning of the lecture was "many composers" -- so I left myself a little leeway there. I did not say "all composers." And I said, "often not in every piece." However, the truth is -- and I still speak the truth -- most of the time when I look, I find it. Much of the time – I'm going to revise that quickly -- much of the time I do find this pattern being worked out. And my list, my full list of composers is the major ones and the minor ones as well.

So, my list of 7 examples or of 11 examples is by no means exhaustive. Now, so, what I can say about my thesis of the use of the sequence of prime numbers by Western composers is that -- a lot of the time -- that seems to be what they're doing. And when things get really tight, like the end of that Counterpoint in the *Art of Fugue,* it's even tighter, that that's exactly what is happening. Just, by golly, the number of notes in the subject, the number of bars, the places where things happen. So, I am able to convince myself and I'm able to launch my thesis into the world, as I'm doing today. That's true.

Thank you. *[sustained applause]*



57'41" Slide #29

Dropbox-generated transcript, edited by WRB 02/09/2026